

Math 100

Paul T Allen

2026-05-17

Table of contents

Welcome to Math 100!	1
Student Learning Outcomes	3
1. Fractions, Decimals, Percents	3
2. Variables and equations	3
3. Geometry	3
4. Modeling relations	4
5. Linear relations	4
6. Exponential relations	4
I. Computation review	5
1. Review: integer arithmetic	7
1.1. Addition and subtraction	7
1.2. Multiplication	8
1.3. Division	9
1.4. Homework exercises	10
2. Review: fractions	13
2.1. Simplifying fractions	13
2.2. Fractions and division	14
2.3. Adding and subtracting fractions	15
2.4. Multiplying fractions	16
2.5. Dividing fractions	17
2.6. Homework exercises	18
3. Fractions, decimals, percents	23
3.1. Proportion and percent	23
3.2. Computing percents from amounts	25
3.3. Computing amounts from percents	26
3.4. Homework exercises	27
4. Percent change	33
4.1. Absolute and percent change	33
4.2. Percent growth	34
4.3. Homework exercises	35

II. Introduction to equations	37
5. Variables and equations	39
5.1. Constructing equations	40
5.2. Using equations	41
5.3. Solving equations	42
5.4. Homework exercises	43
6. Equivalent equations	47
6.1. Equivalent statements	47
6.2. Equivalent scenarios	48
6.3. Equivalent equations	49
6.4. Equivalent scenarios with variables	50
6.5. Equivalence with fractions	52
6.6. Homework exercises	53
7. Solving by operations	59
7.1. Equal operations preserve equality	59
7.2. Practice with operations	60
7.3. Operations with fractions	61
7.4. Two-operation solutions	62
7.5. Practice with two operation solutions	63
7.6. Homework exercises	65
III. Geometry	69
8. Rectangles and squares	71
8.1. Perimeter	72
8.2. Area	73
8.3. Practice: perimeter and area	74
8.4. Finding a missing dimension	75
8.5. Square roots	78
8.6. Homework exercises	80
9. Circles	83
9.1. Circumference	84
9.2. Area	85
9.3. Practice: circumference and area	86
9.4. Finding a missing dimension	87
9.5. Homework exercises	90
10. Right Triangles and the Pythagorean Theorem	93
10.1. Area of a right triangle	94
10.2. The Pythagorean Theorem	96
10.3. Finding missing sides	99
10.4. Applications	101
10.5. Homework exercises	104

11. Rectangular Boxes	109
11.1. Volume	110
11.2. Surface area	111
11.3. Practice: volume and surface area	112
11.4. Finding a missing dimension	113
11.5. Homework exercises	114
12. Geometry Review	117
IV. Modeling Relations	123
13. Data tables	125
13.1. Data tables from formulas	126
13.2. Data tables from base values and rates	128
13.3. Data tables from percents	129
13.4. Homework exercises	130
14. Data plots	135
14.1. Jordan goes for a walk	135
14.2. Plotting and the Cartesian Plane	136
14.3. Examples from Chapter 13	138
14.4. Homework exercises	143
15. Practice with data tables and plots	149
15.1. Choosing horizontal and vertical scales	149
15.2. More modeling scenarios	153
15.3. Homework exercises	156
V. Linear Relations I	161
16. Linear vs nonlinear relations	163
16.1. Group discussion	168
16.2. Homework exercises	169
17. The linear model framework	175
17.1. Generic linear model	175
17.2. Homework exercises	179
18. Linear model analysis	185
18.1. Interpreting linear formulas	185
18.2. Challenge: determining the rate from data	189
18.3. Homework exercises	190
VI. Linear Relations II	195
19. The concept of slope	197
19.1. General formula for rate	199

Table of contents

19.2. Computing the rate from the plot	200
19.3. Computing slope from a data table	201
19.4. Discussion: We have the slope, what about the base?	205
19.5. Section 19.3 revisited	207
19.6. Homework exercises	211
20. Linear model synthesis	215
20.1. General features of the linear model	215
20.2. Suppose we are given a scenario in words	216
20.3. Suppose we are given the plot	218
20.4. Suppose we are given an equation	220
20.5. Suppose we are given a data table	222
20.6. Homework exercises	226
21. Linear practice	237
21.1. Linear scenarios	237
21.2. Interpreting linear plots	241
21.3. Interpreting linear equations	243
22. Project: Taos Bee Association	247
22.1. Analyzing supply	247
22.2. Analyzing demand	248
22.3. Analyzing the equilibrium	248
22.4. Summarizing your analysis	249
VII. Exponential Models	251
23. The basic exponential model	253
23.1. Discussion: features of an exponential model	257
24. The exponential model framework	259
24.1. Parameters of the exponential model	259
24.2. Discussion: Where do A and R appear?	259
24.3. General formula	260
24.4. Exponential increase and decrease	261
24.5. Discussion: increasing and decreasing models	263
24.6. Reading exponential models from data tables	264
24.7. Homework exercises	265
25. Exponential models with percent change	273
25.1. The percent change framework	274
25.2. Practice	275
25.3. Homework exercises	278
26. Compounded percent change	283
26.1. The compounded percent change framework	286
26.2. Practice	287
26.3. Homework exercises	288

27. Practice with exponential models	291
27.1. Exponential scenarios	291
27.2. Interpreting exponential plots	295
27.3. Interpreting exponential formulas	296
27.4. Reading exponential models from data tables	299
28. Project: Radioactive decay, half-lives, and C-14 dating	301
28.1. Part 1: Iodine-131	301
28.2. Part 2: Cesium-137	302
28.3. Part 3: Carbon-14	303
28.4. Part 4: Exploration	303

Welcome to Math 100!

I'm glad you are here!

This website contains materials for the Math 100 course as taught by Paul T Allen at University of New Mexico - Taos.

Solutions to exercises

Solutions to (most of) the practice problems and exercises are available at website version of these notes. There are two reasons for providing solutions.

- First, I encourage students to check their work, but only *after they have attempted the exercises*. Checking your work is a way to get immediate feedback about whether you are understanding the material.
- Second, it is easy to get tutors or various AI engines to generate solutions. However, those solutions might not be formatted or styled in a way that is appropriate for this course, or might even use methods that students have not yet learned. By providing solutions here, I know that students have access to solutions that are appropriate for the narrative of this course.

Errors and suggestions

Any project of this size is bound to involve errors, including mathematical errors, technical/coding errors, and errors of judgement. If you find an error, or have a suggestion for improving the material, please contact me via my UNM email address.

Acknowledgements and License

The material in this packet was created and curated by Paul T Allen. The course was inspired by curriculum developed by Karon Klipple and Cheryl Heddon. Many thanks to Jordan Alexander, Hollie Salazar, and Lena Sherbakov for discussions and suggestions.

This work was created with the assistance of generative AI.

The material is released under the CC-BY creative commons license.

Student Learning Outcomes

1. Fractions, Decimals, Percents

- 1a. Students can add/subtract/multiply/divide integers and fractions without calculator assistance. Students can simplify expressions using parentheses.
- 1b. Students can convert between fractions (in lowest terms), decimals, and percents.
- 1c. Given the total quantity, students can convert between percent and quantity.
- 1d. Students can convert between percent change and absolute change.

2. Variables and equations

- 2a. Students can convert between a linear algebraic equation and a written/verbal description of a scenario, including identifying variable meaning.
- 2b. Students can convert between equivalent equations involving addition and multiplication of rational numbers.
- 2c. Students can solve one-step equations with rational coefficients using additive or multiplicative inverses.
- 2d. Students can solve two-step equations of the form

$$ax + b = c \quad \text{and} \quad a(x + b) = c.$$

3. Geometry

- 3a. Students can use geometric formulas for area and perimeter of rectangles, solving for one quantity when others are given.
- 3b. Students can use geometric formulas for area and perimeter of squares, solving for one quantity when others are given. This includes the use of square root.
- 3c. Students can use geometric formulas for area and circumference of circles, solving for one quantity when others are given.
- 3d. Students can use Pythagorean Theorem to find one side of a right triangle when the other two are given.
- 3e. Students can use geometric formulas for area and perimeter of right triangles, solving for one quantity when others are given.

4. Modeling relations

- 4a. Students can interpret a data table showing pairs of values for two variables.
- 4b. Students can create a data table from a formula.
- 4c. Students can plot values from a data table on Cartesian plane, including choosing appropriate horizontal and vertical scales.
- 4d. Students can interpret points on the plane in terms of data table context.

5. Linear relations

- 5a. Students can distinguish between linear and nonlinear scenarios.
- 5b. Students can identify “base” and “rate” of linear modeling scenarios and construct corresponding linear models.
- 5c. Given a linear model, students can construct a data table and corresponding plot.
- 5d. Given a linear model and the value of one quantity, students can identify which variable has been provided and solve for the other variable.
- 5e. Given two data points, students can compute the “rate” (slope) of the corresponding linear model, and subsequently deduce the “base”, thus obtaining the equation of the model.
- 5f. Given the graph of a model, students can obtain and interpret the equation of the model.

6. Exponential relations

- 6a. Students can identify “starting amount” and “ratio” for exponential relation and construct corresponding data table and linear model $y = A \cdot R^x$.
- 6b. Given an exponential relation, students can construct data table and corresponding plot.
- 6c. Students can interpret the plot of an exponential relation, computing/estimating the value of one variable if the other is given.

Part I.

Computation review

1. Review: integer arithmetic

We start with a short review of computing with integers.

1.1. Addition and subtraction

Here are two examples:

- $5 + 12 = 17$
- $5 - 12 = -7$

i Subtraction and negative numbers

Subtraction is the same as the addition of the opposite/negative of a number:

- $7 - 4$ is the same as $7 + (-4)$
- $8 - (-5)$ is the same as $8 + 5$

Example 1.1 (Practice: addition and subtraction).

1. $12 + 18 =$
2. $-9 + 15 =$
3. $7 + (-13) =$
4. $-8 + (-12) =$
5. $-14 + 9 =$
6. $11 + (-16) =$
7. $20 - 13 =$
8. $-12 - 7 =$
9. $18 - (-9) =$
10. $-15 - (-8) =$

1. Review: integer arithmetic

1.2. Multiplication

Multiplication is a short-cut for repeatedly adding. For example:

$$4 \times 3 = 3 + 3 + 3 + 3 = 12.$$

Here is another example:

$$3 \times (-5) = (-5) + (-5) + (-5) = -15.$$

i Multiplication notation

We have three different ways to write multiplication. Here are three different ways to write “three times four”.

- Cross notation: $3 \times 4 = 12$
- Dot notation: $3 \cdot 4 = 12$
- Neighbor notation: $3(4) = 12$

Example 1.2 (Practice: multiplication).

1. $7 \cdot 9 =$

2. $(-5)(8) =$

3. $12 \cdot (-4) =$

4. $6(11) =$

5. $(-8) \cdot (-3) =$

6. $9(-7) =$

7. $15 \cdot 4 =$

8. $(-6)(-9) =$

1.3. Division

Division is the un-doing of multiplication. Sometimes, numbers divide evenly:

$$48 \div 6 = 8.$$

Other times, there is a remainder:

$$26 \div 6 = 4 \text{ R } 2.$$

Example 1.3 (Practice: division).

1. $72 \div 8 =$

2. $45 \div 5 =$

3. $63 \div 9 =$

4. $56 \div 7 =$

5. $84 \div 12 =$

6. $90 \div 6 =$

7. $50 \div 7 =$

8. $65 \div 8 =$

9. $58 \div 6 =$

10. $77 \div 9 =$

1. Review: integer arithmetic

1.4. Homework exercises

Exercise 1.1. Compute.

1. $8 + 15$

2. $-7 + 12$

3. $9 + (-14)$

4. $-6 + (-11)$

5. $-13 + 8$

6. $5 + (-9)$

7. $12 - 7$

8. $-8 - 5$

9. $15 - (-6)$

10. $-10 - (-4)$

Exercise 1.2. Compute.

1. $7 \cdot 9$

2. $(-5)(8)$

3. $12 \cdot (-4)$

4. $6(11)$

5. $(-8) \cdot (-3)$

6. $9(-7)$

7. $15 \cdot 4$

8. $(-6)(-9)$

9. $(-11)(5)$

10. $8 \cdot (-6)$

1. Review: integer arithmetic

Exercise 1.3. Compute.

1. $96 \div 8$

2. $54 \div 6$

3. $81 \div 9$

4. $64 \div 8$

5. $108 \div 12$

6. $75 \div 5$

7. $62 \div 7$

8. $73 \div 9$

9. $85 \div 11$

10. $59 \div 8$

2. Review: fractions

2.1. Simplifying fractions

One interpretation of fractions is this: divide whole objects into parts, and then take a certain number of those parts.

For example, the quantity $\frac{3}{4}$ pound of beans means:

- divide a pound of beans into 4 parts
- consider only 3 of those parts

We can generate equivalent fractions by grouping and cancelling:

$$\frac{6}{8} = \frac{3 \cdot 2}{4 \cdot 2} = \frac{3}{4}.$$

Example 2.1 (Practice: simplifying fractions).

1. $\frac{12}{18} =$

2. $\frac{15}{25} =$

3. $\frac{20}{32} =$

4. $\frac{24}{36} =$

5. $\frac{30}{45} =$

6. $\frac{16}{28} =$

2.2. Fractions and division

We can also interpret a fraction as division.

For example, suppose we have 3 pounds of beans and want to take one fourth of the total. Here are three equivalent ways to write a fourth of three:

$$\frac{1}{4}(3) = \frac{3}{4} = 3 \div 4.$$

Example 2.2 (Practice: fractions and division).

1. $\frac{1}{5}(2) =$

2. $\frac{7}{3} =$

3. $5 \div 8 =$

4. $\frac{1}{6}(5) =$

5. $\frac{9}{4} =$

6. $7 \div 10 =$

7. $\frac{1}{3}(8) =$

8. $\frac{11}{5} =$

9. $4 \div 9 =$

10. $\frac{1}{7}(6) =$

11. $\frac{13}{6} =$

12. $8 \div 11 =$

2.3. Adding and subtracting fractions

In order to add or subtract fractions, we must write each fraction using a common denominator.

For example:

$$\begin{aligned}\frac{2}{3} - \frac{3}{4} &= \frac{6}{12} - \frac{9}{12} \\ &= -\frac{3}{12} \\ &= -\frac{1}{4}.\end{aligned}$$

i Negative fractions

We can write negative fractions in several ways:

$$\frac{-2}{3} = \frac{2}{-3} = -\frac{2}{3}.$$

My preference is to use the last version, but all three are correct (and equivalent).

Example 2.3 (Practice: adding and subtracting fractions).

1. $\frac{2}{5} + \frac{1}{3} =$

2. $\frac{3}{4} - \frac{1}{6} =$

3. $-\frac{5}{8} + \frac{3}{4} =$

4. $\frac{7}{10} - \frac{2}{5} =$

5. $-\frac{2}{3} - \frac{1}{4} =$

6. $\frac{5}{6} + \left(-\frac{1}{2}\right) =$

7. $-\frac{3}{7} + \frac{2}{3} =$

8. $\frac{4}{9} - \frac{5}{6} =$

2.4. Multiplying fractions

Multiplying by a whole number is a shortcut for addition:

$$3 \cdot \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3 \cdot 2}{5} = \frac{6}{5}.$$

Multiplying by a fraction is a combination of addition and division:

$$\frac{3}{4} \cdot \frac{2}{5} = \frac{3 \cdot 2}{4 \cdot 5} = \frac{6}{20} = \frac{3}{10}.$$

Example 2.4 (Practice: multiplying fractions).

1. $5 \cdot \frac{3}{7} =$

2. $\frac{2}{3} \cdot \frac{4}{5} =$

3. $-6 \cdot \frac{2}{9} =$

4. $\frac{5}{8} \cdot \frac{3}{4} =$

5. $\frac{7}{10} \cdot \left(-\frac{2}{3}\right) =$

6. $4 \cdot \frac{5}{6} =$

7. $\frac{3}{5} \cdot \frac{7}{9} =$

8. $-\frac{4}{7} \cdot \frac{5}{8} =$

9. $\frac{9}{11} \cdot \frac{2}{3} =$

10. $\frac{5}{6} \cdot \left(-\frac{3}{4}\right) =$

2.5. Dividing fractions

In order to divide fractions, we convert into a multiplication problem.

For example

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12} = \frac{5}{6}.$$

Example 2.5 (Practice: dividing fractions).

1. $\frac{3}{4} \div \frac{2}{5} =$

2. $\frac{5}{6} \div \frac{3}{8} =$

3. $-\frac{7}{10} \div \frac{2}{3} =$

4. $\frac{4}{9} \div \frac{5}{6} =$

5. $\frac{8}{15} \div \left(-\frac{4}{5}\right) =$

6. $-\frac{9}{11} \div \frac{3}{7} =$

7. $\frac{7}{12} \div \frac{5}{8} =$

8. $\frac{5}{9} \div \left(-\frac{10}{13}\right) =$

2. Review: fractions

2.6. Homework exercises

Exercise 2.1. Simplify each fraction.

1. $\frac{14}{21}$

2. $\frac{18}{27}$

3. $\frac{25}{40}$

4. $\frac{28}{42}$

5. $\frac{32}{48}$

6. $\frac{35}{50}$

7. $\frac{22}{33}$

8. $\frac{27}{36}$

Exercise 2.2. Write each expression in the other two equivalent forms.

1. $\frac{1}{4}(3)$

2. $\frac{5}{7}$

3. $6 \div 11$

4. $\frac{1}{9}(4)$

5. $\frac{8}{5}$

6. $9 \div 13$

7. $\frac{1}{8}(7)$

8. $\frac{10}{3}$

9. $5 \div 12$

10. $\frac{1}{6}(11)$

2. Review: fractions

Exercise 2.3. Compute. Simplify your answer if possible.

1. $\frac{1}{4} + \frac{2}{5}$

2. $\frac{5}{6} - \frac{1}{3}$

3. $-\frac{3}{10} + \frac{2}{5}$

4. $\frac{7}{12} - \frac{1}{4}$

5. $-\frac{5}{9} - \frac{1}{6}$

6. $\frac{3}{8} + \left(-\frac{1}{4}\right)$

7. $-\frac{2}{5} + \frac{3}{7}$

8. $\frac{5}{12} - \frac{7}{8}$

9. $\frac{4}{15} + \frac{2}{9}$

10. $-\frac{3}{8} - \frac{5}{12}$

Exercise 2.4. Compute. Express your answer as an improper fraction in simplified form.

1. $7 \cdot \frac{2}{5}$

2. $\frac{3}{4} \cdot \frac{5}{6}$

3. $-8 \cdot \frac{3}{10}$

4. $\frac{4}{9} \cdot \frac{6}{7}$

5. $\frac{5}{12} \cdot \left(-\frac{3}{8}\right)$

6. $6 \cdot \frac{7}{9}$

7. $\frac{2}{7} \cdot \frac{5}{11}$

8. $-\frac{8}{9} \cdot \frac{3}{5}$

9. $\frac{7}{8} \cdot \frac{4}{9}$

10. $\frac{9}{10} \cdot \left(-\frac{5}{6}\right)$

2. Review: fractions

Exercise 2.5. Compute. Express your answer as an improper fraction in simplified form.

1. $\frac{2}{3} \div \frac{4}{7}$

2. $\frac{7}{8} \div \frac{3}{5}$

3. $-\frac{5}{9} \div \frac{2}{7}$

4. $\frac{6}{11} \div \frac{4}{9}$

5. $\frac{9}{14} \div \left(-\frac{3}{8}\right)$

6. $-\frac{7}{10} \div \frac{5}{12}$

7. $\frac{8}{15} \div \frac{6}{7}$

8. $\frac{11}{16} \div \left(-\frac{5}{9}\right)$

9. $\frac{4}{13} \div \frac{8}{15}$

10. $-\frac{9}{20} \div \frac{3}{10}$

3. Fractions, decimals, percents

3.1. Proportion and percent

Another interpretation of fractions is as a *proportion*.

For example, the statement “ $3/4$ of the people in Taos County like dogs” means that in a typical group of 4 people, three of them will like dogs.

We can also represent proportion as a *percent*, which tells us how many compared to a group size of 100.

- Since $\frac{3}{4} = \frac{75}{100}$, the proportion $3/4$ is equivalent to 75%.

Finally, we can represent fraction as a decimal number.

- We have $\frac{3}{4} = \frac{75}{100} = 0.75$.

In summary, we have $\frac{3}{4} = 75\% = 0.75$

Example 3.1 (Practice: converting fractions to decimals and percents).

1. $\frac{1}{4} =$
2. $\frac{3}{5} =$
3. $\frac{7}{8} =$
4. $\frac{2}{3} =$
5. $\frac{5}{6} =$
6. $\frac{3}{10} =$
7. $\frac{5}{4} =$
8. $\frac{7}{5} =$

3. *Fractions, decimals, percents*

Example 3.2 (Practice: converting percents to fractions).

1. $25\% =$

2. $40\% =$

3. $75\% =$

4. $60\% =$

5. $3.5\% =$

6. $8.25\% =$

Example 3.3 (Practice: converting decimals to percents and fractions).

1. $0.4 =$

2. $0.7 =$

3. $0.125 =$

4. $0.625 =$

5. $1.09 =$

6. $0.375 =$

3.2. Computing percents from amounts

The UNM-Taos Institutional Research Office gathers and computes various data about students attending UNM-Taos. The following problems use data from the Spring 2026 Census Snapshot.

In Spring 2026, UNM-Taos had 1321 students. Of these students, 717 were continuing students (who had attended a previous semester). The percent of continuing students was

$$\frac{\text{number of continuing students}}{\text{number of total students}} = \frac{717}{1321} \approx 0.54 = 54\%.$$

Example 3.4 (Practice: computing percents from amounts).

1. There were 50 transfer students. What percent is this?
2. There were 57 first-year students. What percent is this?
3. There were 933 who consider UNM-Taos as their “home campus” (as opposed to dual-enrollment students or students from other UNM campuses who are taking classes here). What percent is this?

3.4. Homework exercises

Exercise 3.1. Convert each fraction to a decimal and to a percent.

1. $\frac{1}{2}$

2. $\frac{4}{5}$

3. $\frac{3}{8}$

4. $\frac{5}{9}$

5. $\frac{7}{10}$

6. $\frac{1}{3}$

7. $\frac{2}{5}$

8. $\frac{9}{4}$

9. $\frac{11}{5}$

10. $\frac{8}{3}$

3. *Fractions, decimals, percents*

Exercise 3.2. Convert each percent to a fraction. Simplify your answer.

1. 30%

2. 50%

3. 20%

4. 65%

5. 45%

6. 70%

7. 15%

8. 90%

9. 12.5%

10. 37.5%

Exercise 3.3. Convert each decimal to a percent and to a fraction. Do not simplify the fraction.

1. 0.6

2. 0.3

3. 0.875

4. 0.225

5. 1.15

6. 0.625

7. 0.475

8. 2.25

9. 0.325

10. 1.85

3. *Fractions, decimals, percents*

Exercise 3.4. According to the 2020 census, Taos County had a total population of 34,489 people. Use this information to answer the following questions. Round your answers to two decimal places.

1. There were 17,430 people classified as Hispanic residents. What percent is this?
2. There were 13,706 people classified as White (non-Hispanic) residents. What percent is this?
3. There were 2,108 people classified as American Indian/Alaska Native residents. What percent is this?
4. Of the 15,747 households, 3,514 had children under the age of 18 living with them. What percent of households is this?
5. There were 20,904 housing units in the county, of which 5,157 were vacant. What percent of housing units were vacant?
6. Of the 34,489 residents, 18,833 lived in rural areas. What percent lived in rural areas?

Exercise 3.5. According to the 2020 census, New Mexico had a total population of 2,117,522 people. Use this information to answer the following questions. Round your answers to the nearest whole number.

1. 47.7% of the residents were classified as Hispanic. How many residents is this?
2. 36.5% of the residents were classified as White (non-Hispanic). How many residents is this?
3. 9.5% of the residents were classified as American Indian/Alaska Native. How many residents is this?
4. 18.5% of the residents were ages 65 and older. How many residents is this?
5. 76.1% of the residents were ages 18 and older. How many residents is this?
6. 1.8% of the residents were classified as Black or African American. How many residents is this?
7. 2.8% of the residents identified as two or more races. How many residents is this?

4. Percent change

4.1. Absolute and percent change

According to the Spring 2026 Census Snapshot, in Spring 2025 UNM-Taos had 881 home students and in Spring 2026, there were 933 home students.

The *absolute change* in the number of home students is computed by taking the ending value and subtracting the starting value:

$$933 - 881 = 52 \text{ students.}$$

The *percent change* in the number of home students is computed by viewing the absolute change as a proportion of the starting number:

$$\frac{52 \text{ students}}{881 \text{ students}} = 0.059 = 5.9$$

Example 4.1 (Practice: absolute and percent change).

1. In 2010, Taos County had a population of 32,937. In 2020, the population was 34,489. Compute the absolute change and the percent change.
2. This year, Los Luceros Historic Site has 51 Churro sheep. Last year, they had 43 sheep. Compute the absolute change and the percent change.
3. The Town of Taos had a population of 5,716 in 2010. In 2020, the population was 6,474. Compute the absolute change and the percent change.
4. This week the UNM Taos Cafe sold 23 frito pies. Last week they sold 37 frito pies. Compute the absolute change and the percent change.

4. Percent change

4.2. Percent growth

A local credit union is offering 1-year certificates of deposit (CDs) with an annual percentage rate of 2.5%. This means that if we deposit \$500 in a CD account now, the value in one year will be 2.5% higher.

One way to compute the value after one year is to first compute the change in the value:

$$\text{change in value} = 2.5\% \text{ of } \$500 = 0.025 \cdot 500 = \$12.50.$$

Thus after one year we have

$$\begin{aligned} \text{new value} &= \text{old value} + \text{change in value} \\ &= \$500 + \$12.50 \\ &= \$512.50. \end{aligned}$$

A second way to compute the value after one year is to combine the percents:

$$\begin{aligned} \text{new value} &= 100\% \text{ of the original amount} + 2.5\% \text{ of the original amount} \\ &= 102.5\% \text{ of the original amount.} \end{aligned}$$

This means that the new value is equal to

$$102.5\% \text{ of } \$500 = 1.025 \cdot 500 = \$512.50.$$

While both methods are equally valid, you are encouraged to use the second method wherever possible.

Example 4.2 (Practice: percent growth).

1. The Town of Taos currently has a population of 6,474. Demographers project a 5% increase over the next decade. What will the population be after this increase?
2. A rancher purchases a truck for \$45,000. The truck is expected to lose 12% of its value during the first year. What will the truck's value be after one year?
3. UNM-Taos currently has 1,321 students enrolled. The administration hopes for an 8% increase in enrollment next year. If this happens, how many students will be enrolled?
4. A small business in Taos had revenue of \$125,000 last year. Due to economic conditions, they expect a 6% decrease in revenue this year. What will their revenue be after this decrease?

4.3. Homework exercises

Exercise 4.1. For each scenario, compute the absolute change and the percent change. Round percent changes to two decimal places.

1. In 2010, New Mexico had a population of 2,059,179. In 2020, the population was 2,117,522. Compute the absolute change and the percent change.
2. Bernalillo County had a population of 662,564 in 2010. In 2020, the population was 676,444. Compute the absolute change and the percent change.
3. This year, a Taos gallery sold 145 pieces of artwork. Last year, they sold 128 pieces. Compute the absolute change and the percent change.
4. In 2020, Santa Fe County had a population of 154,823. In 2010, the population was 144,170. Compute the absolute change and the percent change.
5. A local farm stand sold 89 pounds of green chile this week. Last week they sold 112 pounds. Compute the absolute change and the percent change.
6. Rio Arriba County had a population of 40,246 in 2010. In 2020, the population was 40,363. Compute the absolute change and the percent change.
7. The Taos Ski Valley reported 3,250 skier visits this month. Last month they had 2,890 visits. Compute the absolute change and the percent change.
8. Doña Ana County had a population of 209,233 in 2010. In 2020, the population was 218,195. Compute the absolute change and the percent change.

4. Percent change

Exercise 4.2. For each scenario, compute the resulting value after the percent change. Round to the nearest whole number when appropriate.

1. Taos County currently has a population of 34,489. Demographers project a 3% increase over the next decade. What will the population be after this increase?
2. A restaurant in Taos purchases kitchen equipment for \$28,000. The equipment is expected to lose 15% of its value during the first year. What will the equipment's value be after one year?
3. A local nonprofit had 450 volunteers last year. They hope for a 10% increase in volunteers this year. If this happens, how many volunteers will they have?
4. Los Luceros Historic Site had 8,500 visitors last year. Due to road construction, they expect a 7% decrease in visitors this year. What will their visitor count be after this decrease?
5. The median household income in Taos County is \$58,950. Economists project a 4% increase over the next five years. What will the median household income be after this increase?
6. A Taos art gallery had sales of \$185,000 last year. Due to increased tourism, they expect a 12% increase in sales this year. What will their sales be after this increase?
7. A sheep ranch currently has 240 sheep. Due to drought conditions, the rancher expects an 8% decrease in the herd this year. How many sheep will remain after this decrease?
8. New Mexico had a population of 2,117,522 in 2020. If the population grows by 2% over the next decade, what will the population be?

Part II.

Introduction to equations

5. Variables and equations

In this section we introduce one of the key concepts of algebra: *variables*. The purpose of variables is to be able to write down a particular computation, and manipulate that computation, in a situation where we don't know yet what number we want to put into the computation. The variable is a placeholder for the number(s) that we might want to put in... it's a placeholder for a number that might *vary*. Typically, we use single letters for variables.

Example 5.1. Paul is addicted to the cranberry oatmeal cookie at KoKo, and is interesting in calculating his monthly cookie budget. Currently, cranberry oatmeal cookies cost \$3 each. This means that

$$\begin{aligned} \text{Paul's total monthly cost} \\ &= \$3 \cdot (\text{the number of cookies Paul eats each month}). \end{aligned}$$

We introduce the following variables:

- x is the number of cookies Paul eats each month,
- y is the monthly cost of Paul's cookie habit.

In terms of these variables, we have

$$y = 3 \cdot x.$$

This mathematical object is called an *equation* because it tells us that two things (y and $3 \cdot x$) are always equal.

i Note

Rather than write $3 \cdot x$, it is customary to simply write $3x$ for the number three multiplied by the variable x .

Remember that multiplication is simply lazy addition, and so

$$3x = 3 \cdot x = x + x + x.$$

Using this shortcut notation, the equation in Example 5.1 is simply $y = 3x$.

5.1. Constructing equations

Example 5.2 (Practice: constructing equations). For each of the following scenarios, define the variables and then construct the requested equation that relates them.

1. Each day, Lucas always does 5 more pushups than Hollie. Construct an equation that related the Lucas' pushup count to Hollie's.
2. Metta-the-dog eats two cups of kibble each day. She is going to stay over at a friend's house, and needs to have her kibble packed for her. Construct an equation relating the number of days she will stay to the amount of kibble that must be packed.
3. A typical American uses 90 gallons of water per day in their home. A small town is planning for their water system. Construct an equation relating the population of the town and the total amount of home water use by the residents.
4. Karen always drives 5 miles per hour over the speed limit. Write an equation relating the speed limit and her actual speed.

5.3. Solving equations

i Vocabulary: solving for a variable

Once we have specified the value of one variable, the resulting equation only has one other variable. The process of finding a value for this other variable is called *solving* for the variable. For instance, in the second part of Example 5.3, we were given a value for y and we solved for x .

Example 5.5 (Practice: solving equations). Use the equations that you constructed in Example 5.2 to address the following.

- 1a. Lucas did 23 pushups. How many pushups did Hollie do?

- 1b. Hollie did 18 pushups. How many pushups did Lucas do?

- 2a. Metta-the-dog will stay for 5 days. How much kibble must be packed?

- 2b. 14 cups of kibble were packed. How many days will Metta stay?

- 3a. The town has a population of 850 people. What is the total amount of home water use by the residents?

- 3b. The total home water use is 7,200 gallons per day. What is the population of the town?

- 4a. The speed limit is 35 mph. What is Karen's actual speed?

- 4b. Karen is driving 60 mph. What is the speed limit?

5.4. Homework exercises

Exercise 5.1. For each of the following scenarios, define the variables and then construct the requested equation that relates them.

1. A recipe calls for 3 times as much flour as sugar. Write an equation relating the amount of sugar to the amount of flour.
2. Maria is 7 years older than her sister Ana. Write an equation relating Ana's age to Maria's age.
3. A parking garage charges \$4 per hour. Write an equation relating the number of hours parked to the total cost.
4. The temperature in Taos is always 15 degrees cooler than in Albuquerque. Write an equation relating the Albuquerque temperature to the Taos temperature.
5. A construction worker earns \$25 per hour. Write an equation relating the number of hours worked to the total earnings.
6. Tomas always runs 2 miles more than Sarah. Write an equation relating Sarah's distance to Tom's distance.
7. To convert feet to inches, you multiply by 12. Write an equation relating feet to inches.
8. A store marks up the wholesale price of items by adding \$8. Write an equation relating the wholesale price to the retail price.

5. *Variables and equations*

Exercise 5.2. Solve each problem using the equation from Exercise 5.1.

1a. The amount of flour needed is 9 cups. How much sugar is needed?

1b. The recipe uses 2 cups of sugar. How much flour is needed?

2a. Maria is 25 years old. How old is Ana?

2b. Ana is 14 years old. How old is Maria?

3a. A customer parked for 6 hours. What is the total cost?

3b. The total cost was \$32. How many hours did the customer park?

4a. The temperature in Albuquerque is 78 degrees. What is the temperature in Taos?

4b. The temperature in Taos is 55 degrees. What is the temperature in Albuquerque?

5a. A worker earned \$200. How many hours did they work?

5b. A worker worked 9 hours. How much did they earn?

6a. Tom ran 10 miles. How far did Sarah run?

6b. Sarah ran 7 miles. How far did Tom run?

7a. A measurement is 36 inches. How many feet is this?

7b. A measurement is 5 feet. How many inches is this?

8a. The retail price is \$45. What was the wholesale price?

8b. The wholesale price was \$62. What is the retail price?

6. Equivalent equations

6.1. Equivalent statements

i Vocabulary: equivalent statements

We say that the statements

$$12 \div 4 = 3 \quad \text{and} \quad 12 = 4 \cdot 3$$

are *equivalent statements* because they express the same basic fact, but with opposite operations.

Example 6.1 (Practice: equivalent statements). For each, write an equivalent statement using the opposite operation. There might be more than one possible answer.

1. $15 + 7 = 22$

2. $8 \cdot 5 = 40$

3. $30 - 12 = 18$

4. $56 \div 8 = 7$

5. $9 + 14 = 23$

6. $45 \div 5 = 9$

7. $7 \cdot 6 = 42$

8. $25 - 11 = 14$

6. Equivalent equations

6.2. Equivalent scenarios

If Jacob drives 120 miles in two hours, then we can compute his average speed by

$$\frac{120 \text{ miles}}{2 \text{ hours}} = 60 \text{ miles/hour.}$$

An equivalent scenario is that Jordan averages 60 miles per hour over the course of two hours, with the result that the distance traveled is

$$(60 \text{ miles/hour}) \cdot (2 \text{ hours}) = 120 \text{ miles.}$$

Example 6.2 (Practice: equivalent scenarios). For each scenario, write an equivalent scenario using the opposite operation.

1. Maria runs 15 miles in 3 hours. Her average speed is $\frac{15 \text{ miles}}{3 \text{ hours}} = 5 \text{ miles/hour}$.

2. A worker earns \$25 per hour and works for 8 hours.

The total earnings are $(25 \text{ dollars/hour}) \cdot (8 \text{ hours}) = 200 \text{ dollars}$.

3. A recipe uses 12 cups of flour to make 4 batches of cookies. The amount of flour per batch is $\frac{12 \text{ cups}}{4 \text{ batches}} = 3 \text{ cups/batch}$.

4. Water flows from a hose at 6 gallons per minute for 10 minutes. The total amount of water is $(6 \text{ gallons/minute}) \cdot (10 \text{ minutes}) = 60 \text{ gallons}$.

5. Rachel travels by bicycle 80 kilometers in 4 hours.

Her average speed is $\frac{80 \text{ km}}{4 \text{ hours}} = 20 \text{ km/hour}$.

6.3. Equivalent equations

It is also possible to have equivalent equations. For simplicity, we focus on equations with only one variable.

The equation $x + 5 = 23$ has two different equivalent equations:

- One option is $5 = 23 - x$.
- Another option is $x = 23 - 5$.

Both of the options are mathematically valid, but the second one is more useful because it helps us deduce the value $x = 18$.

Example 6.3 (Practice: equivalent equations). For each equation find an equivalent equation that allows us to deduce the value of the variable.

1. $x + 7 = 20$

2. $3y = 45$

3. $m - 8 = 15$

4. $12 = 4n$

5. $18 = p + 9$

6. $56 = 7k$

7. $w - 12 = 30$

8. $5t = 65$

6. Equivalent equations

6.4. Equivalent scenarios with variables

Example 6.4. Currently, Hail Creek is selling regular gasoline at \$3.85 per gallon. I have \$20 to spend on gas, but I don't know how many gallons of gasoline that will be. I let x represent the number of gallons. I know that

$$\text{total price} = \$3.85 \cdot \text{number of gallons}$$

and so my equation is

$$20 = 3.85x.$$

This is equivalent to the equation

$$\frac{20}{3.85} = x.$$

Using my calculator, I see that $x \approx 5.19$, which means I can purchase 5.19 gallons of gas.

Example 6.5 (Practice: equivalent scenarios with variables).

1. A painter charges \$45 per hour. Maria has a budget of \$270 for painting her living room. Let h represent the number of hours the painter can work. Write an equation and solve for h .

2. Ground beef costs \$6.50 per pound at the grocery store. Kevin wants to spend \$32.50 on ground beef. Let p represent the number of pounds he can buy. Write an equation and solve for p .

6.4. Equivalent scenarios with variables

3. A plumber charges \$85 per hour. A homeowner has \$425 available to pay for plumbing repairs. Let h represent the number of hours the plumber can work. Write an equation and solve for h .
4. Maria has some money saved. After adding \$35 to her savings, she has \$127 total. Let s represent her original savings amount. Write an equation and solve for s .
5. Firewood is sold for \$225 per cord. A family has \$675 budgeted for firewood. Let c represent the number of cords they can buy. Write an equation and solve for c .
6. After spending some money on groceries, Tom has \$43 left from his original \$150. Let g represent the amount he spent on groceries. Write an equation and solve for g .
7. A landscaper charges \$55 per hour. A client has a budget of \$330 for landscaping work. Let h represent the number of hours the landscaper can work. Write an equation and solve for h .

6. Equivalent equations

6.5. Equivalence with fractions

Suppose we want to create an equivalent equation for $\frac{2}{3}x = 8$. Since the original equation involves multiplication of $\frac{2}{3}$ and x , the equivalent equation will involve division:

$$x = 8 \div \frac{2}{3}.$$

But we can convert division by a fraction into multiplication! So another version of our equivalent equation is

$$x = 8 \cdot \frac{3}{2}.$$

Example 6.6 (Practice: equivalence with fractions). For each equation below, construct an equivalent equation that involves multiplication.

1. $\frac{3}{4}x = 12$

2. $\frac{2}{5}y = 10$

3. $18 = \frac{5}{6}m$

4. $\frac{4}{7}n = 20$

5. $15 = \frac{3}{8}p$

6. $\frac{5}{9}w = 25$

6.6. Homework exercises

Exercise 6.1. Write an equivalent statement using the opposite operation.

1. $18 + 9 = 27$

2. $6 \cdot 7 = 42$

3. $35 - 14 = 21$

4. $72 \div 9 = 8$

5. $13 + 19 = 32$

6. $63 \div 7 = 9$

7. $8 \cdot 9 = 72$

8. $50 - 23 = 27$

9. $12 \cdot 5 = 60$

10. $81 \div 9 = 9$

6. *Equivalent equations*

Exercise 6.2. For each scenario, write an equivalent scenario using the opposite operation.

1. A train travels 240 miles in 4 hours. The average speed is $\frac{240 \text{ miles}}{4 \text{ hours}} = 60 \text{ miles/hour}$.
2. A baker makes cookies at a rate of 36 cookies per hour for 5 hours. The total number of cookies is $(36 \text{ cookies/hour}) \cdot (5 \text{ hours}) = 180 \text{ cookies}$.
3. A garden hose fills a pool with 150 gallons of water in 6 hours. The flow rate is $\frac{150 \text{ gallons}}{6 \text{ hours}} = 25 \text{ gallons/hour}$.
4. A construction worker earns \$28 per hour and works for 7 hours. The total earnings are $(28 \text{ dollars/hour}) \cdot (7 \text{ hours}) = 196 \text{ dollars}$.
5. A student reads 180 pages in 6 hours. The reading rate is $\frac{180 \text{ pages}}{6 \text{ hours}} = 30 \text{ pages/hour}$.
6. A factory produces 75 items per hour for 8 hours. The total production is $(75 \text{ items/hour}) \cdot (8 \text{ hours}) = 600 \text{ items}$.
7. A car uses 56 gallons of gas to travel 896 miles. The fuel efficiency is $\frac{896 \text{ miles}}{56 \text{ gallons}} = 16 \text{ miles/gallon}$.
8. A runner maintains a pace of 8 minutes per mile for 5 miles. The total time is $(8 \text{ minutes/mile}) \cdot (5 \text{ miles}) = 40 \text{ minutes}$.

Exercise 6.3. For each equation find an equivalent equation that allows us to deduce the value of the variable.

1. $x + 9 = 25$

2. $4y = 52$

3. $m - 6 = 19$

4. $18 = 3n$

5. $27 = p + 11$

6. $72 = 8k$

7. $w - 15 = 40$

8. $6t = 84$

9. $35 = r + 14$

10. $9s = 108$

6. *Equivalent equations*

Exercise 6.4. For each scenario, define the variable, write an equation, and solve for the variable.

1. A mechanic charges \$75 per hour. A customer has \$450 available for car repairs. Let h represent the number of hours the mechanic can work. Write an equation and solve for h .
2. Fresh salmon costs \$14.50 per pound at the fish market. A chef wants to spend \$87 on salmon. Let p represent the number of pounds they can buy. Write an equation and solve for p .
3. A tree removal service charges \$180 per tree. A homeowner has a budget of \$900 for tree removal. Let t represent the number of trees they can have removed. Write an equation and solve for t .
4. Sarah has some money in her checking account. After depositing \$250, she has \$815 total. Let b represent her original balance. Write an equation and solve for b .

6.6. Homework exercises

5. Premium coffee beans cost \$16.75 per pound. A coffee shop has \$134 to spend on beans. Let p represent the number of pounds they can buy. Write an equation and solve for p .

6. A personal trainer charges \$65 per session. A client has \$390 budgeted for training. Let s represent the number of sessions they can afford. Write an equation and solve for s .

7. Ceramic tiles cost \$4.25 per tile. A homeowner has \$255 to spend on tiles. Let t represent the number of tiles they can purchase. Write an equation and solve for t .

8. After paying some bills, Carlos has \$185 remaining from his original \$520. Let b represent the amount he paid in bills. Write an equation and solve for b .

9. Organic strawberries cost \$5.50 per basket. A customer has \$27.50 to spend. Let b represent the number of baskets they can buy. Write an equation and solve for b .

10. An electrician charges \$95 per hour. A business has \$570 budgeted for electrical work. Let h represent the number of hours the electrician can work. Write an equation and solve for h .

6. *Equivalent equations*

Exercise 6.5. For each equation, create an equivalent equation and solve for the variable.

1. $\frac{2}{3}x = 18$

2. $\frac{3}{5}y = 21$

3. $24 = \frac{4}{7}m$

4. $\frac{5}{8}n = 30$

5. $16 = \frac{2}{9}p$

6. $\frac{7}{10}w = 35$

7. $\frac{3}{4}k = 27$

8. $40 = \frac{5}{6}t$

9. $\frac{4}{9}r = 36$

10. $28 = \frac{7}{12}s$

7. Solving by operations

In this section we introduce the *operation framework*, which is a method for converting an equation into an equivalent equation.

7.1. Equal operations preserve equality

The key idea is this: if two things are equal, and we do the same operation to both them, then the results are also equal.

For example:

- Start with $2 \cdot 3$ and 6, which are equal.
- Perform the operation $+5$ to both.
- The result is $2 \cdot 3 + 5$ and $6 + 5$, which are also equal.

Here is one way to write this process in mathematical symbols

$$\begin{array}{r} 2 \cdot 3 = 6 \\ +5 \quad +5 \\ 2 \cdot 3 + 5 = 6 + 5 \end{array}$$

The previous example is a bit silly, so let's look at another example that shows how this concept helps us solve equations.

Here is the process in words:

- We start with the equation $x + 3 = 22$.
- We perform the operation -3 to both $x + 3$ and to 22.
- The result is that $x + 3 - 3$ is equal to $22 - 3$.
- But $+3 - 3$ cancels itself out, and we are left with $x = 22 - 3$.
- This means that $x = 19$.

Here is one way to write this in math symbols.

$$\begin{array}{r} x + 3 = 22 \\ -3 \quad -3 \\ x + 3 - 3 = 22 - 3 \\ x = 22 - 3 \\ x = 19 \end{array}$$

7. Solving by operations

7.2. Practice with operations

Example 7.1. For each equation, decide what operation would improve the equation. Apply that operation and then simplify to find the value of x .

1. $5x = 35$

2. $x + 9 = 24$

3. $-3x = 21$

4. $x - 7 = 15$

5. $8x = 72$

6. $x + 12 = 30$

7. $-6x = 42$

8. $4x = -28$

9. $x - 11 = 19$

10. $7x = 91$

7.3. Operations with fractions

In order to solve the equation

$$\frac{2}{3}x = 6$$

we can apply the operation “multiply by $\frac{3}{2}$ ” to both sides. If we do this, the left side cleans up nicely:

$$\begin{aligned}\frac{3}{2} \cdot \frac{2}{3}x &= \frac{3}{2} \cdot 6 \\ x &= 9\end{aligned}$$

Example 7.2 (Practice: operations with fractions).

1. $\frac{2}{3}x = 18$

2. $\frac{5}{4}x = \frac{15}{8}$

3. $-\frac{3}{7}x = 12$

4. $\frac{4}{5}x = 20$

5. $\frac{3}{8}x = \frac{9}{16}$

6. $-\frac{2}{5}x = \frac{6}{7}$

7. $\frac{7}{9}x = 14$

8. $-\frac{5}{6}x = -\frac{10}{3}$

7. Solving by operations

7.4. Two-operation solutions

Some equations require two operations in order to solve for the unknown. For example, consider the equation

$$5x - 3 = 17.$$

There are two operations on the left side that we must un-do:

- The variable x is multiplied by 5, which can be un-done with $\div 5$.
- The quantity $5x$ has 3 subtracted, which can be un-donw with $+3$.

Which operation to do first? The key to deciding is to keep in mind that the operation applies to the entirety of the two sides.

- If we apply the operation $\div 5$, then the result is

$$(5x - 3) \div 5 = 17 \div 5$$

which we can also write as

$$\frac{5x - 3}{5} = \frac{17}{5}.$$

- If we apply the operation $+3$, then the result is

$$(5x - 3) + 3 = 17 + 3$$

which we can also write as

$$5x - 3 + 3 = 17 + 3.$$

Which of these operations makes the left side of the equation look simpler?

We choose the second option. Thus our plan is to first add 3 and then divide by 5. Here is the full solution:

$$5x - 3 = 17 \quad \text{add 3}$$

$$5x - 3 + 3 = 17 + 3 \quad \text{tidy both sides}$$

$$5x = 20 \quad \text{divide by 5}$$

$$\frac{5x}{5} = \frac{20}{5} \quad \text{more cleaning!}$$

$$x = 4$$

i Which operation to do first?

When deciding which operation to do first, it is important to keep in mind the order of operations. The operations we choose are designed to reverse the operations being done to the variable, and so must happen in the reverse order.

In the previous example, the left side $5x - 3$ means first multiply by 5, then subtract 3. And so to un-do that, we must first add 3 and then divide by 5.

7.5. Practice with two operation solutions

Example 7.3. First decide which operations you will do, and in what order. Then use the operations to solve for x .

1. $3x + 7 = 25$

2. $4(x + 5) = 32$

3. $\frac{x + 3}{2} = 9$

4. $6x - 11 = 31$

5. $5(x - 4) = 35$

6. $\frac{x - 7}{3} = 5$

7. Solving by operations

7. $-2x + 9 = 3$

8. $7(x + 2) = 49$

9. $\frac{x + 8}{4} = 6$

10. $8x - 15 = 41$

11. $3(x - 6) = 21$

12. $\frac{x - 5}{6} = 4$

7.6. Homework exercises

Exercise 7.1. For each equation, decide what operation would improve the equation. Apply that operation to both sides and then simplify to find the value of x .

1. $6x = 48$

2. $x + 11 = 35$

3. $-4x = 32$

4. $x - 9 = 18$

5. $9x = 81$

6. $x + 15 = 42$

7. $-7x = 56$

8. $5x = -40$

9. $x - 13 = 27$

10. $8x = 96$

7. Solving by operations

Exercise 7.2. For each equation, decide what operation would improve the equation. Apply that operation to both sides and then simplify to find the value of x .

1. $\frac{3}{5}x = 15$

2. $\frac{7}{4}x = \frac{21}{8}$

3. $-\frac{4}{9}x = 16$

4. $\frac{x}{5} = 8$

5. $\frac{5}{6}x = 25$

6. $x + \frac{2}{3} = \frac{5}{6}$

7. $-\frac{3}{8}x = \frac{9}{4}$

8. $\frac{x}{7} = 12$

9. $-\frac{7}{10}x = -\frac{14}{5}$

10. $x + \frac{3}{4} = \frac{7}{8}$

Exercise 7.3. For each equation, decide which operations to apply and in what order. Show all steps to find the value of x .

1. $4x + 9 = 37$

2. $5(x + 3) = 40$

3. $\frac{x + 6}{3} = 8$

4. $7x - 12 = 23$

5. $6(x - 5) = 42$

6. $\frac{x - 4}{5} = 7$

7. $-3x + 14 = 5$

8. $8(x + 4) = 64$

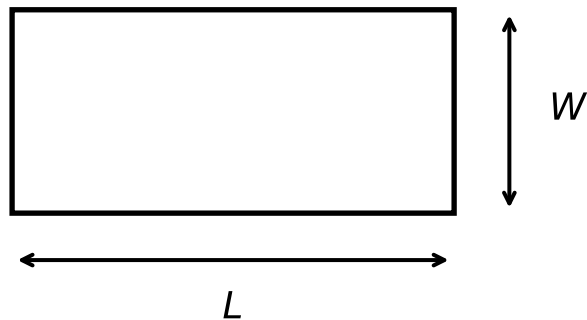
9. $\frac{x + 10}{6} = 5$

10. $9x - 18 = 54$

Part III.
Geometry

8. Rectangles and squares

A *rectangle* is a flat shape with four sides and four right angles. Its two dimensions are the *length* L and the *width* W .



We have two ways to measure the size of a rectangle.

- **Perimeter** (P) is the distance all the way around the outside of the rectangle — like the length of fencing needed to enclose a field.
- **Area** (A) is the number of unit squares needed to tile the inside of the rectangle — like the number of floor tiles needed to cover a room.

Perimeter is measured in units of length (feet, meters). Area is measured in square units (square feet, square meters).

8. Rectangles and squares

8.1. Perimeter

A rectangle has two sides of length L and two sides of length W , so the perimeter is

$$\text{perimeter} = \text{length} + \text{width} + \text{length} + \text{width} = 2 \cdot \text{length} + 2 \cdot \text{width}$$

$$P = 2L + 2W$$

Example 8.1. The Taos Inn has a conference room that is 14 feet long and 10 feet wide. How much decorative trim is needed to go around the entire perimeter of the room?

$$P = 2(14) + 2(10) = 28 + 20 = 48 \text{ feet.}$$

i Squares

A *square* is a special rectangle where $L = W$. If we call the common side length S , the perimeter formula simplifies:

$$P = 2S + 2S = 4S.$$

For example, a square patio with $S = 11$ feet has perimeter $P = 4(11) = 44$ feet.

Example 8.2 (Activity: Marking off park spaces). The town of Red River is laying out two new park spaces.

- **Space A** is a rectangular picnic area, 20 feet long and 12 feet wide.
- **Space B** is a square performance stage with side length 15 feet.

1. How much rope is needed to mark the boundary of Space A?

2. How much rope is needed to mark the boundary of Space B?

8.2. Area

The area of a rectangle is the length multiplied by the width:

$$\text{area} = \text{length} \times \text{width}$$

$$A = L \cdot W$$

Example 8.3. The conference room at the Taos Inn is 14 feet long and 10 feet wide. How much carpet is needed to cover the floor?

$$A = 14 \cdot 10 = 140 \text{ square feet.}$$

i Squares

For a square with side length S , the area formula simplifies:

$$A = S \cdot S = S^2.$$

For example, a square tile with $S = 4$ inches has area $A = 4^2 = 16$ square inches.

Example 8.4 (Activity: Covering the park spaces). Recall the two park spaces from Example 8.2.

1. How many square feet of sod are needed to cover Space A?
2. How many square feet of paving stones are needed for Space B?
3. What is the total area of both spaces combined?

8. Rectangles and squares

8.3. Practice: perimeter and area

Example 8.5 (Activity: Computing perimeter and area). Complete the following table. The first row is completed for you as a reference.

L	W	$P = 2L + 2W$	$A = L \cdot W$
14 ft	10 ft	48 ft	140 sq ft
9 ft	5 ft		
20 ft	8 ft		
6 ft	6 ft		
7 ft	3 ft		
12 ft	12 ft		
4.5 ft	8 ft		

Note: rows 4 and 6 are squares.

8.4. Finding a missing dimension

So far we have computed P and A when both L and W are given. We can also run this process in reverse: if we know the area or perimeter and one dimension, we can set up an equation and solve for the missing dimension. This is the same equation-solving process from the previous chapter, now applied to geometry.

Example 8.6. A storage shed near Taos has a rectangular floor that is 18 feet wide. The total floor area is 270 square feet. How long is the shed?

We use the area formula with $A = 270$ and $W = 18$:

$$270 = L \cdot 18$$

Dividing both sides by 18:

$$L = 270 \div 18 = 15 \text{ feet.}$$

Example 8.7. A rancher near Mora wants to fence a rectangular alfalfa field. The field is 200 feet long, and there is 740 feet of fencing available. How wide can the field be?

We use the perimeter formula with $P = 740$ and $L = 200$:

$$740 = 2(200) + 2W \Rightarrow 740 = 400 + 2W \Rightarrow 340 = 2W \Rightarrow W = 170 \text{ feet.}$$

8. Rectangles and squares

Example 8.8 (Activity: Rectangle problems). For each problem, identify which formula to use, set up the equation, and solve.

1. A classroom at UNM-Taos is 32 feet long. The room requires 864 square feet of new flooring. How wide is the classroom?
2. Maria is planting a rectangular vegetable garden at the Taos Farmers Market. She has 52 feet of fencing and wants the garden to be 16 feet long. How wide can the garden be?
3. A rancher near Cimarron wants to build a rectangular horse corral. The corral will be 45 feet wide, and 200 feet of fencing is available. How long can the corral be?
4. A square tile used at a Taos Pueblo art market has an area of 81 square inches. What is the side length of the tile? (Hint: what number times itself equals 81?)

Example 8.9 (Activity: Missing dimensions). Complete the following table. Some entries are given; find the rest. For the last two rows the rectangle is a square ($L = W = S$): find S first, then complete the row.

L	W	$P = 2L + 2W$	$A = L \cdot W$
8	?		56
?	5		60
15	?	46	
?	6	40	
S	S		64
S	S	36	

8.5. Square roots

In the missing dimension problems above, the last two rows involved a square where we knew the area A and needed to find the side length S . This means solving $S^2 = A$ — finding a number that, when multiplied by itself, gives A .

This operation is called taking the *square root* of A , written \sqrt{A} :

$$\sqrt{A} = S \quad \text{means} \quad S \cdot S = A.$$

Example 8.10. $\sqrt{64} = 8$, because $8 \cdot 8 = 64$.

$\sqrt{36} = 6$, because $6 \cdot 6 = 36$.

$\sqrt{100} = 10$, because $10 \cdot 10 = 100$.

The table below lists the square roots of the first several *perfect squares* — numbers whose square root is a whole number.

A	\sqrt{A}	A	\sqrt{A}
1	1	49	7
4	2	64	8
9	3	81	9
16	4	100	10
25	5	121	11
36	6	144	12

For areas that are not perfect squares, \sqrt{A} is not a whole number. In those cases, use the $\sqrt{\quad}$ button on your calculator.

For example, a square with area 50 square feet has side length $S = \sqrt{50} \approx 7.07$ feet.

Example 8.11 (Activity: Finding side lengths from area). For each square, find the side length S .

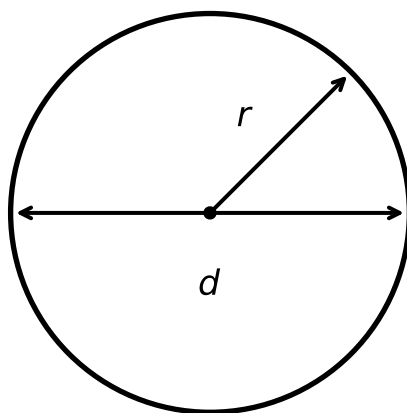
1. A square has area 49 square feet. What is S ?
2. A square has area 144 square meters. What is S ?
3. A square courtyard has area 225 square feet. What is S ? (Hint: try $S = 15$.)
4. A square patio has area 75 square feet. What is S ? Use a calculator and round to two decimal places.

Exercise 8.2. Complete the following table. Some entries are given; find the rest. For the last two rows the rectangle is a square ($L = W = S$): find S first, then complete the row.

L	W	$P = 2L + 2W$	$A = L \cdot W$
11	4		
6	?		48
?	7		84
20	?	62	
?	9	50	
S	S		100
S	S	44	

9. Circles

A *circle* is a round, flat shape where every point on the edge is the same distance from the center. That distance is the *radius*, labeled r . The *diameter* d is the distance across the circle through the center: $d = 2r$.



We have two ways to measure the size of a circle.

- **Circumference** (C) is the distance all the way around the outside — the circle’s version of perimeter.
- **Area** (A) is the amount of flat space inside the circle.

Circumference is measured in units of length (feet, meters). Area is measured in square units (square feet, square meters).

Both formulas involve a special number called π (the Greek letter “pi”).

i The number π

The ratio of circumference to diameter is the same for every circle, no matter the size. That ratio is π :

$$\pi = \frac{C}{d} \approx 3.14159 \dots$$

π is not an exact fraction — its decimal expansion continues forever without repeating. For calculations, use the π button on your calculator. For rough estimates by hand, $\pi \approx 3.14$ is sufficient.

9. Circles

9.1. Circumference

Since $\pi = C/d$ and $d = 2r$, we can rearrange to get the circumference formula:

$$C = \pi d = 2\pi r.$$

Example 9.1. The fountain at Taos Plaza is circular with a radius of 5 feet. What is the circumference of the fountain?

$$C = 2\pi(5) = 10\pi \approx 31.4 \text{ feet.}$$

Example 9.2 (Activity: Edging two garden beds). The UNM-Taos community garden has two circular planting beds.

- **Bed A** has a radius of 4 feet.
- **Bed B** has a diameter of 14 feet.

Each bed will be bordered with a metal edging strip.

1. How many feet of edging does Bed A require?

2. What is the radius of Bed B?

3. How many feet of edging does Bed B require?

9. Circles

9.3. Practice: circumference and area

Example 9.5 (Activity: Computing circumference and area). Complete the table. Round all answers to two decimal places. For the last two rows, find r from the given diameter first.

r	$d = 2r$	$C = 2\pi r$	$A = \pi r^2$
3 ft			
5 ft			
8 ft			
	12 ft		
	9 ft		

9.4. Finding a missing dimension

So far we have computed C and A when the radius is known. We can also reverse this: if the circumference or area is given, we can solve for the radius.

Finding r from circumference. Start from $C = 2\pi r$ and solve for r .

Example 9.6. A center-pivot irrigation system near Taos traces a circular path with a circumference of approximately 628 feet. How long is the pivot arm (the radius of the circle)?

$$628 = 2\pi r \Rightarrow r = \frac{628}{2\pi} \approx 100 \text{ feet.}$$

Finding r from area. Start from $A = \pi r^2$ and solve for r . The first step is to divide both sides by π , leaving r^2 alone. Then take the square root of both sides.

Example 9.7. A circular rug at a Taos gallery has an area of approximately 28.3 square feet. What is the radius of the rug?

$$28.3 = \pi r^2 \Rightarrow r^2 = \frac{28.3}{\pi} \approx 9 \Rightarrow r = \sqrt{9} = 3 \text{ feet.}$$

9. Circles

Example 9.8 (Activity: Circle problems). For each problem, set up an equation and solve. Round to two decimal places where needed.

1. A circular hot tub at a Taos resort has a radius of 3.5 feet. What is the area of the cover needed?
2. A decorative porthole window at a Santa Fe inn is circular with a diameter of 2 feet. What is the circumference of the window frame?
3. A circular reflecting pool at the Millicent Rogers Museum has a circumference of approximately 47.1 feet. What is the radius of the pool?
4. A circular dining table at a Taos restaurant has a surface area of approximately 12.57 square feet. What is the radius of the table?

Example 9.9 (Activity: Missing dimensions). Complete the table. Round all answers to two decimal places.

r	$d = 2r$	$C = 2\pi r$	$A = \pi r^2$
		25.13 ft	
			113.10 sq ft
	62.83 ft		
			78.54 sq ft

9.5. Homework exercises

Exercise 9.1. For each problem, set up an equation and solve. Round to two decimal places where needed.

1. A circular swimming pool at a Taos resort has a diameter of 18 feet. What is the circumference of the pool? What is the area of the pool's surface?
2. The observation area at a Taos Mountain overlook is a circular deck with a radius of 12 feet. What is the area of the deck?
3. A circular hay bale has a circumference of approximately 94.2 inches. What is the diameter of the bale?
4. A circular stained-glass window at a church in Ranchos de Taos has an area of approximately 176.7 square inches. What is the radius of the window? What is the diameter?

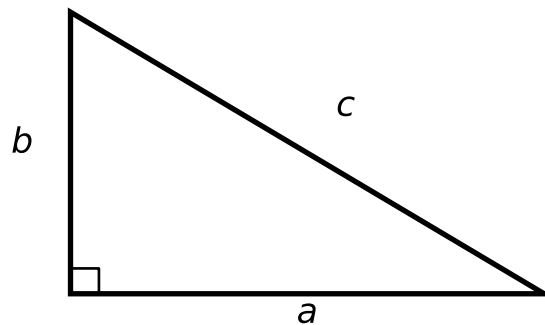
Exercise 9.2. Complete the table. Round all answers to two decimal places. For rows where the diameter is given, find r first.

r	$d = 2r$	$C = 2\pi r$	$A = \pi r^2$
4 ft			
	16 ft		
		43.98 ft	
			254.47 sq ft
11 ft			
		18.85 ft	

10. Right Triangles and the Pythagorean Theorem

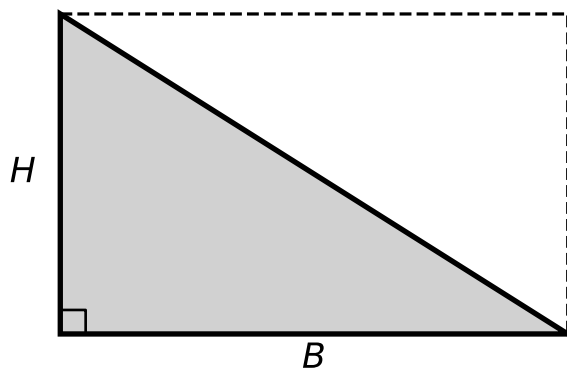
i Vocabulary: right triangles

A *right triangle* is a triangle with one right angle (a 90° corner). The two sides that form the right angle are the *legs*. The side opposite the right angle — always the longest side — is the *hypotenuse*.



10.1. Area of a right triangle

The key observation is that every right triangle fits exactly inside a rectangle. The two legs of the triangle become the sides of the rectangle, and the remaining corner of the rectangle completes the shape.



Because the diagonal cuts the rectangle into two identical triangles, the right triangle takes up exactly half the rectangle's area.

Example 10.1. A right triangle has legs of length 6 feet and 4 feet. The surrounding rectangle has area $6 \cdot 4 = 24$ square feet. The triangle is exactly half the rectangle, so its area is $\frac{1}{2} \cdot 24 = 12$ square feet.

In general, if the two legs of a right triangle have lengths B and H :

$$A = \frac{1}{2} \cdot B \cdot H.$$

i Base, height, and legs

For a right triangle, the two legs are perpendicular to each other, so either leg can serve as the base and the other as the height.

Example 10.2 (Activity: Area of right triangles). Draw a picture of each triangle and find the area.

1. Legs: 3 ft and 4 ft.

2. Legs: 5 ft and 12 ft.

3. Legs: 8 ft and 15 ft.

4. A triangular section of a garden plot at the Taos Farmers Market has legs of 9 feet and 6 feet. What is the area of the plot?

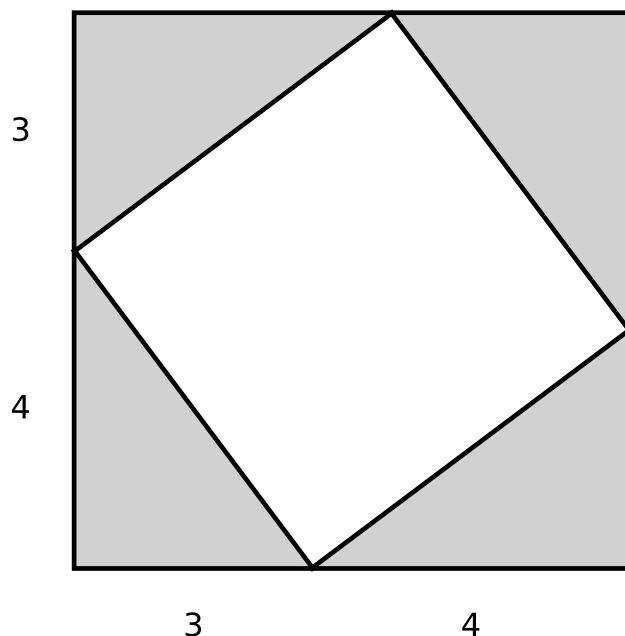
10.2. The Pythagorean Theorem

The legs and hypotenuse of a right triangle are connected by a remarkable relationship. We will discover this relationship through a geometric argument.

Discovering the theorem

Start with a large square whose side length equals the sum of the two legs of our right triangle. Place four copies of the right triangle in the corners of the square, each pointing inward. The four triangles leave a smaller tilted square in the middle.

The figure below shows this construction for a right triangle with legs 3 and 4.

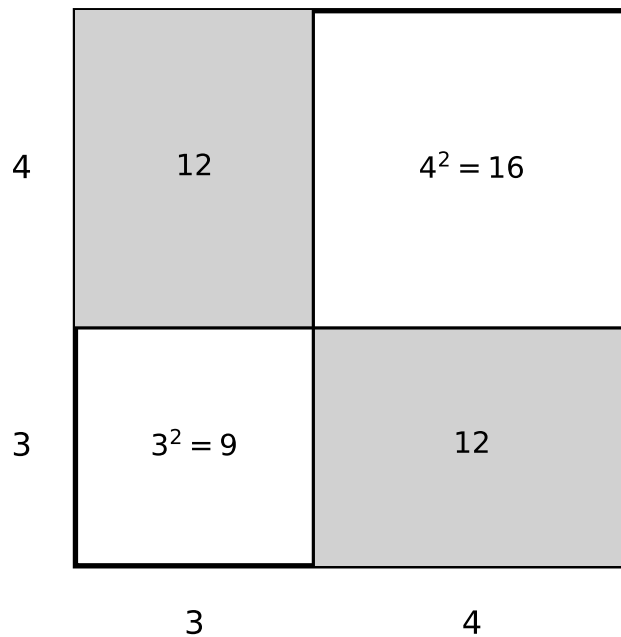


Example 10.3 (Activity: Discovering the Pythagorean Theorem). Use the figure above. The large square has side $3 + 4 = 7$.

1. What is the area of the large square?
2. Each shaded right triangle has legs 3 and 4. What is the area of one triangle? What is the combined area of all four triangles?
3. Subtract the four triangles from the large square. What is the area of the inner tilted square?
4. What is the side length of the inner square? (Use a square root.)

5. The side of the inner square is the hypotenuse of each right triangle. Compute $3^2 + 4^2$. What do you notice?

But why does $3^2 + 4^2$ give exactly the same answer as c^2 ? A second way of subdividing the same big square explains why. Draw a vertical line at distance 3 from the left and a horizontal line at distance 3 from the bottom.



This subdivision shows the big square as two smaller squares ($3^2 = 9$ and $4^2 = 16$, shown white) and two rectangles (each $3 \cdot 4 = 12$, shown gray). Both diagrams include the same gray area — four triangles in the first diagram, two rectangles in the second — each totaling 24. Since the total area is the same in both diagrams:

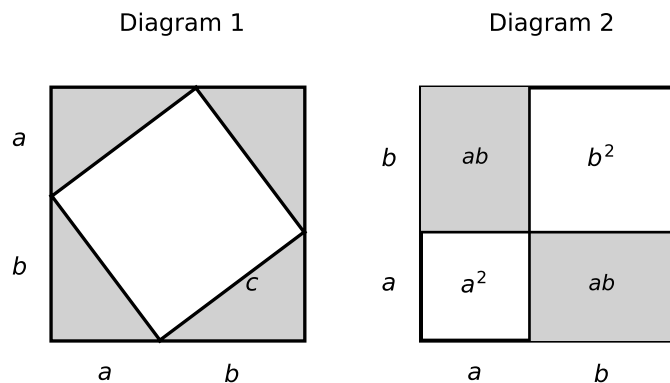
$$c^2 + 24 = 9 + 16 + 24 \quad \Rightarrow \quad c^2 = 9 + 16 = 3^2 + 4^2.$$

10. Right Triangles and the Pythagorean Theorem

The general argument

The same two diagrams work for any right triangle with legs a and b and hypotenuse c .

- **Diagram 1** shows the tilted inner square (area $c^2 + 2ab$);
- **Diagram 2** shows the axis-aligned subdivision (area $a^2 + b^2 + 2ab$).



Both diagrams describe the same large square, so their areas are equal:

$$c^2 + 2ab = a^2 + b^2 + 2ab.$$

Subtracting $2ab$ from both sides gives the **Pythagorean Theorem**:

$$\boxed{a^2 + b^2 = c^2}$$

where a and b are the legs and c is the hypotenuse of any right triangle.

10.3. Finding missing sides

Finding the hypotenuse

If both legs are known, square them, add, and take the square root.

Example 10.4. A right triangle has legs of 5 feet and 12 feet. Find the hypotenuse.

$$\begin{aligned}5^2 + 12^2 &= c^2 \\25 + 144 &= c^2 \\169 &= c^2 \\c &= \sqrt{169} = 13 \text{ feet.}\end{aligned}$$

Finding a leg

If the hypotenuse and one leg are known, rearrange $a^2 + b^2 = c^2$ to isolate the unknown leg.

Example 10.5. A right triangle has a hypotenuse of 17 feet and one leg of 8 feet. Find the other leg.

$$\begin{aligned}8^2 + b^2 &= 17^2 \\64 + b^2 &= 289 \\b^2 &= 225 \\b &= \sqrt{225} = 15 \text{ feet.}\end{aligned}$$

Example 10.6 (Activity: Finding missing sides). Diagrams are provided for problems 1 and 2. Draw your own diagram for problems 3–5.

Diagram 1

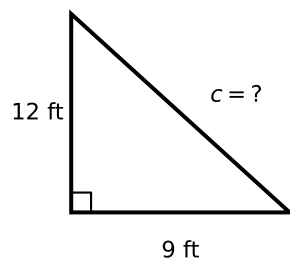
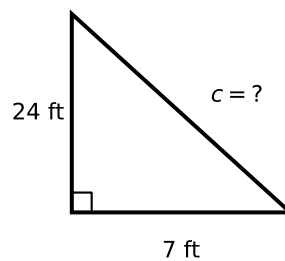


Diagram 2



1. Find the hypotenuse of the triangle in Diagram 1.
2. Find the hypotenuse of the triangle in Diagram 2.

10. *Right Triangles and the Pythagorean Theorem*

3. Hypotenuse: 10 ft. One leg: 6 ft. Find the other leg.

4. Hypotenuse: 13 ft. One leg: 5 ft. Find the other leg.

5. Legs: 4 ft and 7 ft. Find the hypotenuse. Round to two decimal places.

10.4. Applications

Example 10.7. A 13-foot ladder leans against the wall of a building in Taos. The base of the ladder is 5 feet from the wall. How high up the wall does the ladder reach? The ladder, wall, and ground form a right triangle with hypotenuse 13 and one leg 5.

$$5^2 + h^2 = 13^2$$

$$25 + h^2 = 169$$

$$h^2 = 144$$

$$h = 12 \text{ feet.}$$

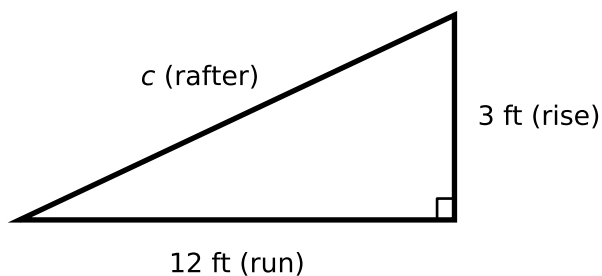
Example 10.8. A rectangular field near Ranchos de Taos measures 30 feet by 40 feet. How long is the diagonal path from one corner to the opposite corner?

$$c^2 = 30^2 + 40^2$$

$$c^2 = 900 + 1600 = 2500$$

$$c = \sqrt{2500} = 50 \text{ feet.}$$

Example 10.9. A house in Taos has a roof with a 12:3 pitch — for every 12 feet of horizontal run, the roof rises 3 feet. The house is 24 feet wide, so each side of the roof has a horizontal run of 12 feet. How long is a rafter on one side of the roof? The rafter, the rise, and the horizontal run form a right triangle with legs 12 and 3.



$$c^2 = 12^2 + 3^2$$

$$c^2 = 144 + 9 = 153$$

$$c = \sqrt{153} \approx 12.4 \text{ feet.}$$

3. A support wire is attached to the top of a 15-foot antenna and anchored to the ground 9 feet from the base. How long is the wire?

4. A hiker near Taos walks 7 miles east and then 7 miles north. How far is the hiker from the starting point? Round to two decimal places.

10.5. Homework exercises

Exercise 10.1. Find the missing side of each right triangle. Round to two decimal places where needed.

Leg a	Leg b	Hypotenuse c
6	8	
9		15
	40	41
10	10	
11	60	
20		29

Exercise 10.2. For each problem, draw a diagram, identify the right triangle, and solve. Round to two decimal places where needed.

1. A 10-foot ladder leans against a wall in Taos. The base of the ladder is 6 feet from the wall. How high up the wall does the ladder reach?

2. A rectangular plot at the UNM-Taos campus measures 20 feet by 48 feet. What is the length of the diagonal?

10. Right Triangles and the Pythagorean Theorem

3. A rancher near Cimarron stretches a wire from the top of an 8-foot fence post diagonally to the ground. The wire is 10 feet long. How far from the base of the post is the wire anchored?

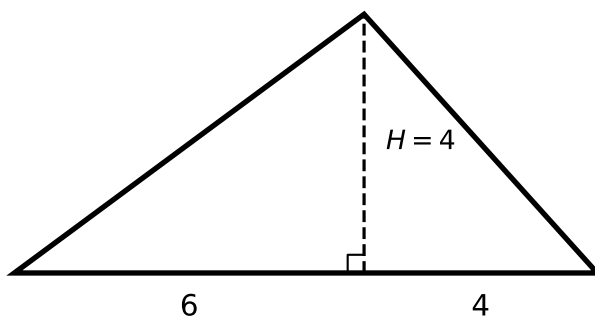
4. Two hikers start from the same point. One walks 5 miles south and the other walks 12 miles east. How far apart are the hikers?

Exercise 10.3. Challenge: Area of any triangle

The formula $A = \frac{1}{2}B \cdot H$ works for all triangles, not just right triangles — as long as H is the *height*, meaning the perpendicular distance from the base to the opposite vertex.

For a right triangle, the legs are already perpendicular, so no extra work is needed. For other triangles, we must find H by drawing an altitude.

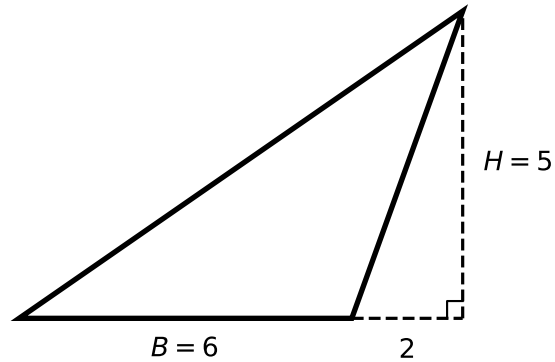
Part 1. The triangle below has base $B = 10$ and the altitude from the opposite vertex to the base has length $H = 4$. The foot of the altitude divides the base into segments of length 6 and 4, creating two right triangles.



Find the area of each right triangle, then add them to find the total area of the triangle. Does your answer match $\frac{1}{2}(10)(4)$?

10. Right Triangles and the Pythagorean Theorem

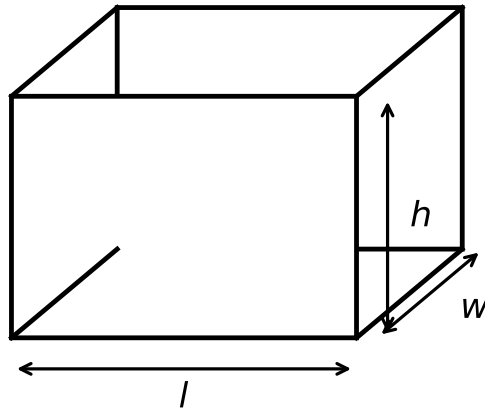
Part 2. Now consider a triangle where the altitude falls *outside* the base. The figure below shows a triangle with base $B = 6$. When the altitude of height $H = 5$ is drawn, its foot falls 2 units beyond the right end of the base, creating a large right triangle (legs 8 and 5) and a small right triangle (legs 2 and 5).



Find the area of each right triangle. Subtract the small from the large to find the area of the original triangle. Does your answer match $\frac{1}{2}(6)(5)$?

11. Rectangular Boxes

A *rectangular box* (also called a *rectangular prism*) is a solid shape with six flat, rectangular faces. Think of a shoebox, a brick, or a room. Every corner is a right angle.



A rectangular box has three dimensions:

- **Length** l — the longest horizontal dimension
- **Width** w — the shorter horizontal dimension
- **Height** h — the vertical dimension

11. Rectangular Boxes

11.1. Volume

Volume measures how much three-dimensional space a solid occupies. It is measured in *cubic units* — cubic inches, cubic feet, cubic centimeters, and so on.

The volume of a rectangular box is:

$$V = l \cdot w \cdot h.$$

You can think of it as stacking layers: each layer has area $l \cdot w$, and there are h layers.

Example 11.1. A storage bin at the UNM-Taos library measures 3 feet long, 2 feet wide, and 1.5 feet tall. What is the volume of the bin?

$$V = 3 \cdot 2 \cdot 1.5 = 9 \text{ cubic feet.}$$

Example 11.2 (Activity: Computing volume). For each box, find the volume. Include units in your answer.

1. A cardboard shipping box is 18 inches long, 12 inches wide, and 10 inches tall.
2. A raised garden bed at the UNM-Taos community garden is 8 feet long, 4 feet wide, and 1.5 feet deep. How many cubic feet of soil does it hold?
3. A refrigerator interior measures 2.5 feet wide, 2 feet deep, and 5 feet tall. What is the volume?

11.3. Practice: volume and surface area

Example 11.5 (Activity: Computing volume and surface area). Complete the table. Round all answers to two decimal places.

l	w	h	$V = lwh$	$SA = 2lh + 2lw + 2wh$
4 ft	3 ft	2 ft		
10 ft	5 ft	6 ft		
7 in	7 in	7 in		
12 cm	8 cm	3 cm		
6 m	2.5 m	4 m		

11.4. Finding a missing dimension

If the volume and two dimensions are known, we can solve for the third dimension. Start from $V = l \cdot w \cdot h$ and divide both sides by the known dimensions.

Example 11.6. A rectangular planter box has a volume of 60 cubic feet, a length of 10 feet, and a width of 3 feet. What is the height?

$$60 = 10 \cdot 3 \cdot h$$

$$60 = 30h$$

$$h = \frac{60}{30} = 2 \text{ feet.}$$

Example 11.7 (Activity: Finding missing dimensions). For each problem, set up an equation and solve for the missing dimension.

1. A fish tank has a volume of 360 cubic inches. It is 15 inches long and 8 inches wide. What is its height?
2. A sandbox at a Taos park has a volume of 48 cubic feet. It is 8 feet long and 2 feet deep. What is the width?
3. A box of tiles at a Taos hardware store has a volume of 1,200 cubic centimeters. The box is 20 cm long and 10 cm wide. How tall is the box?

Exercise 11.2. Complete the table. Round all answers to two decimal places. For rows where the volume and two dimensions are given, find the missing dimension first.

l	w	h	$V = lwh$	$SA = 2lh + 2lw + 2wh$
5 ft	4 ft	3 ft		
8 ft	6 ft		288 cu ft	
9 in		4 in	180 cu in	
	3 m	5 m	120 cu m	
7 cm	7 cm	7 cm		
12 ft	10 ft	8 ft		

12. Geometry Review

Example 12.1 (Activity: Market tent). The Taos Farmers Market sets up a rectangular tent that is 40 feet long and 15 feet wide. What is the perimeter of the tent's footprint? What is the area of the floor space inside the tent?

12. *Geometry Review*

Example 12.2 (Activity: Reflecting pool). A circular reflecting pool at a Taos resort has a diameter of 20 feet. What is the circumference of the pool? What is the area of the pool's surface?

Example 12.3 (Activity: Garden triangle). A triangular section of a community garden at UNM-Taos has legs of 12 feet and 5 feet. Is the longest side of this triangle exactly 13 feet? Show your work using the Pythagorean theorem.

12. *Geometry Review*

Example 12.4 (Activity: Ranch shed). A storage shed at a Taos ranch is 10 feet long, 8 feet wide, and 7 feet tall. What is the volume of the shed? How many square feet of material are needed to cover all six sides?

Example 12.5 (Activity: Mosaic tile). A circular mosaic tile at a Santa Fe studio has a circumference of approximately 37.70 inches. What is the radius of the tile? What is the area of the tile?

Part IV.

Modeling Relations

13. Data tables

Data tables are an efficient way to organize and present information. In this class, we use data tables that relate two different quantities (also called *variables*).

Example 13.1 (Activity: reading a data table). The following table shows the relationship between the variables

- x is the time, measured in years,
- y is the population of Mora County, measured in people.

x time	y population
1980	4,205
1990	4,264
2000	5,180
2010	4,881
2020	4,189

Data source: Wikipedia

Answer these questions based on the table:

- When $x = 1980$, what is y ?

- When $x = 2000$, what is y ?

- For what value of x is the value of y greater than 4200?

In the next part of this section, we explore three ways to construct data tables.

13.1. Data tables from formulas

One way to construct a data table is using a formula.

Example 13.2 (Activity: data table from a formula). We use the variables

- x is the width of the square, and
- y is the area of the square.

Use this formula to complete the following table.

x width	y area
1	$1^2 = 1$
2	$2^2 = 4$
3	
4	
5	

Then answer the following questions:

- If $x = 3$, what is the value of y ?

- If $x = 5$, what is the value of y ?

- If $y = 49$, what is the value of x ?

Example 13.3 (Activity: data table from a constraint). There are many possible rectangles that have an area of 48 square inches. Complete the following data table giving the length and width of possible rectangles.

x length	y width
1	48
2	
3	
4	
5	
6	
7	
8	

Answer the following questions:

- If $x = 3$, what is y ?

- If $x = 6$, what is y ?

- If $y = 12$, what is x ?

- Is it possible for x and y to have the same value?

13.3. Data tables from percents

Finally, we can compute data tables from percent changes.

Example 13.5 (Activity: data table from percent change). Prices are always going up, and the Rancho Coffee Company is no exception. In 2015, when they opened, a cup of *Talpa House Coffee* cost \$2.00. Each year since then the price has risen by 5%.

The following table uses the variables:

- x is the number of years since 2015,
- y is the price of a cup of *Talpa House Coffee*.

Complete the table:

x years since 2015	y price
0	\$2.00
1	\$2.10
2	\$2.21
3	
4	
5	
6	

Answer the following questions:

- How did we compute that when $x = 2$ then $y = \$2.21$?

- In what year did the price exceed \$2.50?

13.4. Homework exercises

Exercise 13.1. The table below shows the average temperature in Santa Fe for the first 12 days of January 2026. (Source: <https://www.weather.gov/wrh/Climate?wfo=abq>) The table uses the variables

- x the day of the month,
- y the average temperature that day (measured in degrees F).

Day x	Average temperature y
1	43
2	44
3	42
4	43
5	37
6	38
7	38
8	37
9	28
10	24
11	30
12	36

Use the table to answer the following questions:

- When $x = 4$, what is y ?

- When $x = 9$, what is y ?

- What x value corresponds to the largest y value?

- What x value corresponds to the smallest y value?

13. *Data tables*

Exercise 13.3. Construct a table showing the relationship between the variables

- x the width of a square, and
- y the perimeter of the square.

Your table should have 5 rows in it.

Use your table to address these questions:

- If $x = 3$, what is y ?

- If $y = 16$, what is x ?

Exercise 13.4. Rosalie buys a 50 pound bag of dog food. Each day her dogs, named Bruno and Macho, eat a total $1/2$ pound of food.

Make a table that relates the variables

- x the number of days since Rosalie bought the bag of food, and
- y the amount of dog food remaining in the bag.

Your table should have at least 7 rows in it.

Use your table to address these questions:

- If $x = 2$, what is y ?

- If $x = 6$, what is y ?

- If $y = 48$, what is x ?

Challenge: what value of x will correspond to $y = 0$? What does this mean about the bag of food?

13. Data tables

Exercise 13.5. Santiago buys a used pickup truck for \$20,000. Each year, the truck loses 5% of its value. (In business-speak this is called *depreciation*.) Make a table that relates these variables:

- x is the number of years since Santiago bought the truck, and
- y the value of the truck.

Your table should have at least 5 rows in it. The first row should have $x = 0$ and $y = 20,000$.

Answer the following questions:

- When $x = 1$, you should have $y = 19,000$. How did this get computed?

- When $x = 2$, what is y ?

- When $x = 5$, what is y ?

Challenge: After 10 years, what will be the value of the truck?

14. Data plots

Data plots are a way to visualize information that appears in a data table. Let's introduce data plots with the following simple example.

14.1. Jordan goes for a walk

On Saturday mornings, Jordan goes for a long walk. His walking speed is 3 miles per hour.

Let's explore this scenario using these variables:

- x is the time walked (in hours)
- y is the distance walked (in miles)

Description via data table

The following table gives a list of values for x . Fill in the missing values for y .

$x = \text{time}$	$y = \text{distance}$
0	0
1	3
2	6
3	
4	

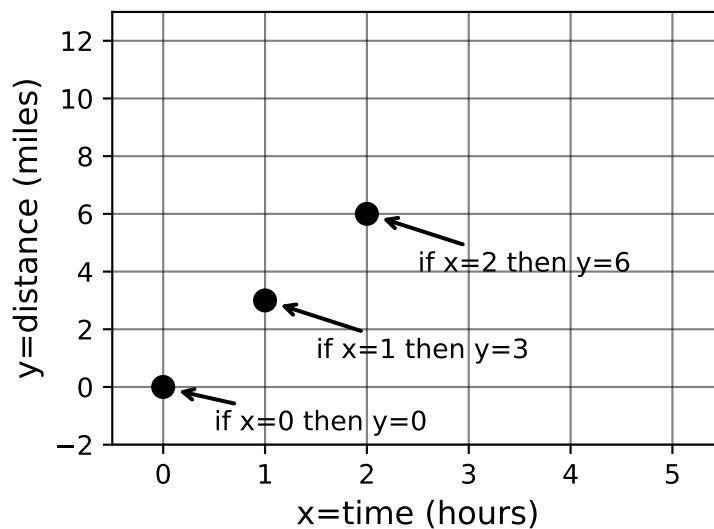
Description via plot

We visually represent the relationship between time and distance using a *plot in the Cartesian plane*. The plot is organized in the following way:

- We make a horizontal number line that represents values of x .
- We make a vertical number line that represents values of y .

14. Data plots

Each row in our table of values represents a pair of values for x and y . Using the horizontal and vertical number lines, we mark a corresponding point in the plot. In the following graphic, the first three data points are indicated.



Example 14.1 (Activity: adding data points). Add the remaining two data points from the table of values to the plot above.

14.2. Plotting and the Cartesian Plane

In the motivating example above, we introduced the idea of plotting using the Cartesian plane. Let's explore the Cartesian plane in more detail.

Axes and Variables

The horizontal and vertical number lines are called *axes*.

While any variable name can be used to describe the axes, there are standard default settings:

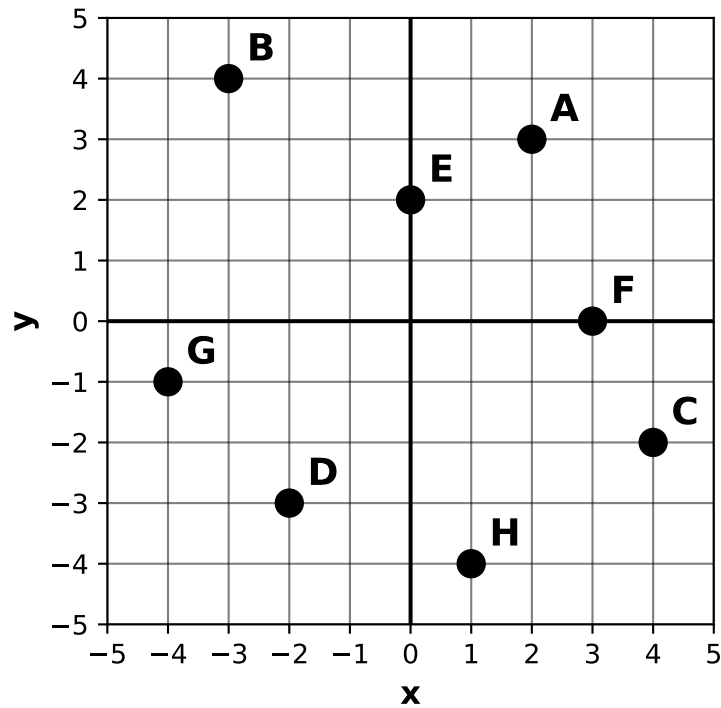
- The default setting for the horizontal axis is the variable x .
- The default setting for the vertical axis is the variable y .

The place where the two axes meet is called the *origin*, and corresponds to both variables being equal to zero.

Ordered pair notation

When identifying a location in the plane, we can specify the x and y values individually. We call these values *coordinates*. For example, location A in the graphic below has coordinates $x = 2$ and $y = 3$.

We use ordered pairs of numbers as a shortcut notation. Thus we can simply say that location A has coordinates $(2, 3)$. Always we list the horizontal value first, followed by the vertical value.



Example 14.2 (Activity: reading coordinates). Use the graphic above to identify the coordinates of each point.

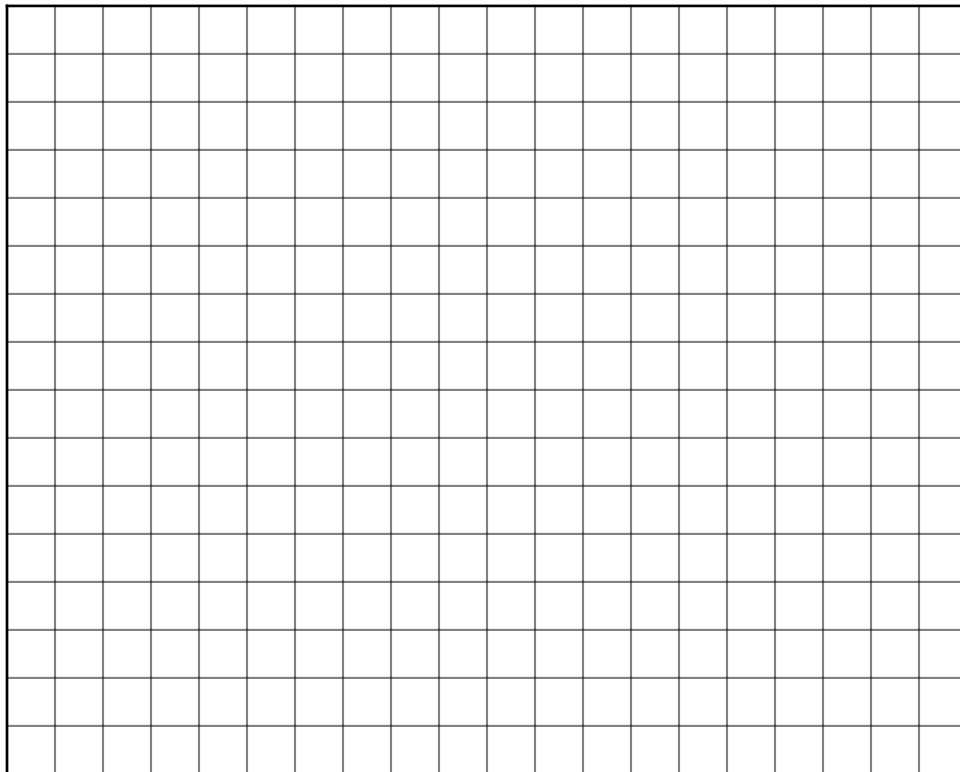
$$\begin{array}{cccc}
 A = (2, 3) & B = & C = & D = \\
 E = & F = & G = & H =
 \end{array}$$

14.3. Examples from Chapter 13

We can make plots of the tables constructed in Chapter 13.

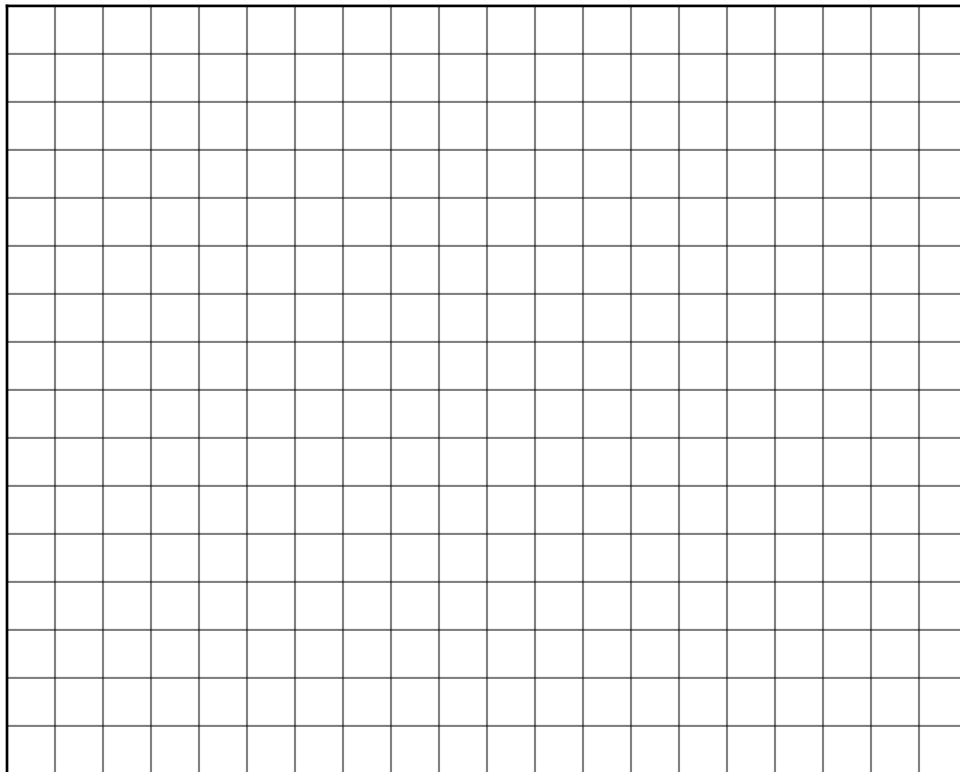
Example 14.3 (Activity: plot Mora County population). Construct a plot showing the data from the table in Example 13.1.

Before you start drawing the plot, think carefully about plotting scale — what are the smallest and largest values you will need for both vertical and horizontal parts of the plot?



Example 14.4 (Activity: plot square area). Construct a plot showing the data from the table in Example 13.2.

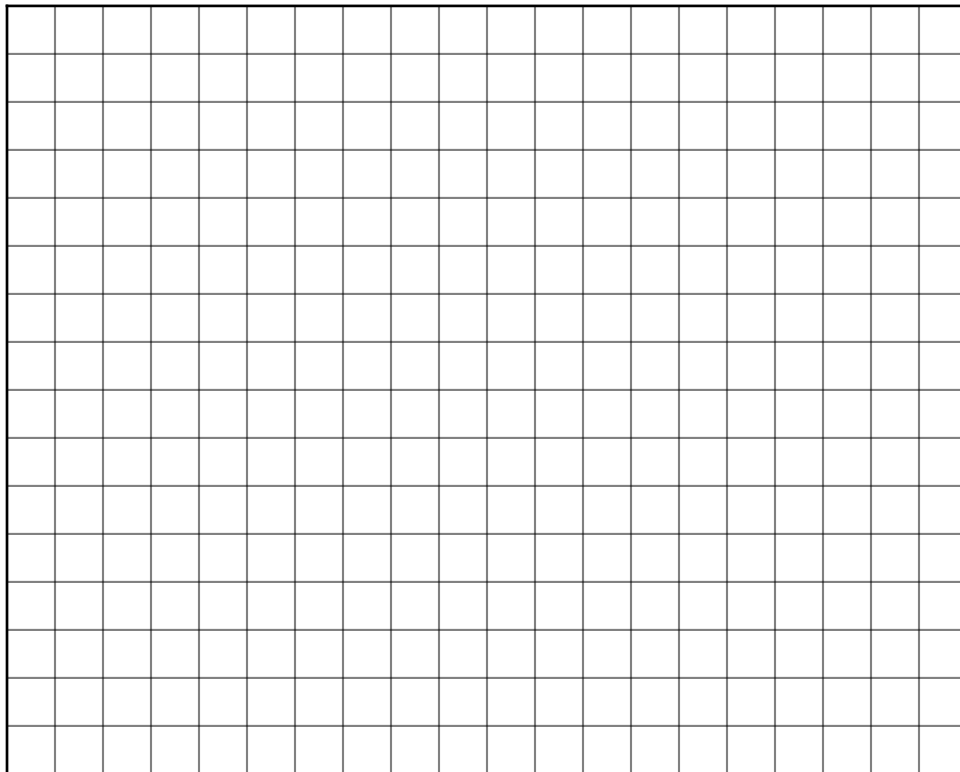
Before you start drawing the plot, think carefully about plotting scale — what are the smallest and largest values you will need for both vertical and horizontal parts of the plot?



14. Data plots

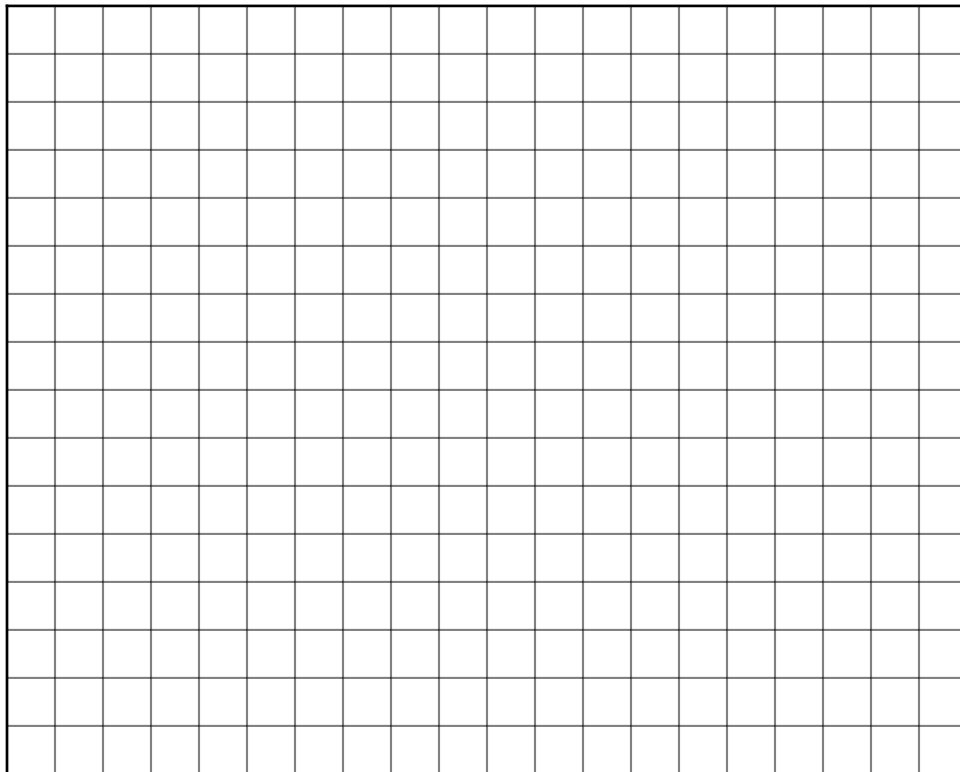
Example 14.5 (Activity: plot rectangle dimensions). Construct a plot showing the data from the table in Example 13.3.

Before you start drawing the plot, think carefully about plotting scale — what are the smallest and largest values you will need for both vertical and horizontal parts of the plot?



Example 14.6 (Activity: plot coffee wages). Construct a plot showing the data from the table in Example 13.4.

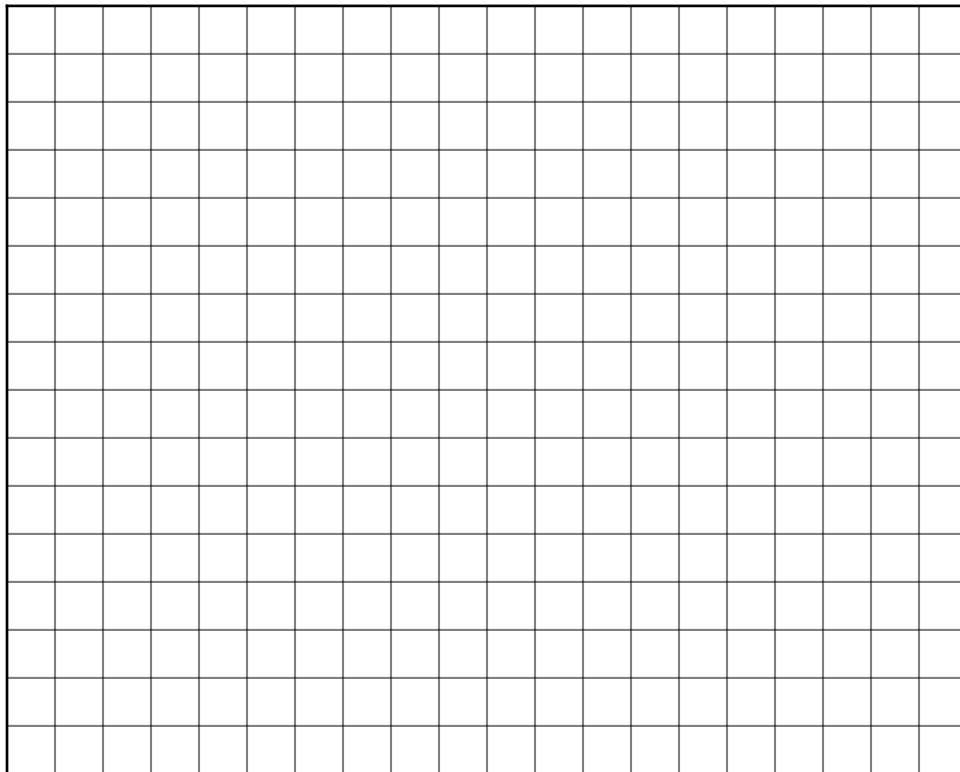
Before you start drawing the plot, think carefully about plotting scale — what are the smallest and largest values you will need for both vertical and horizontal parts of the plot?



14. Data plots

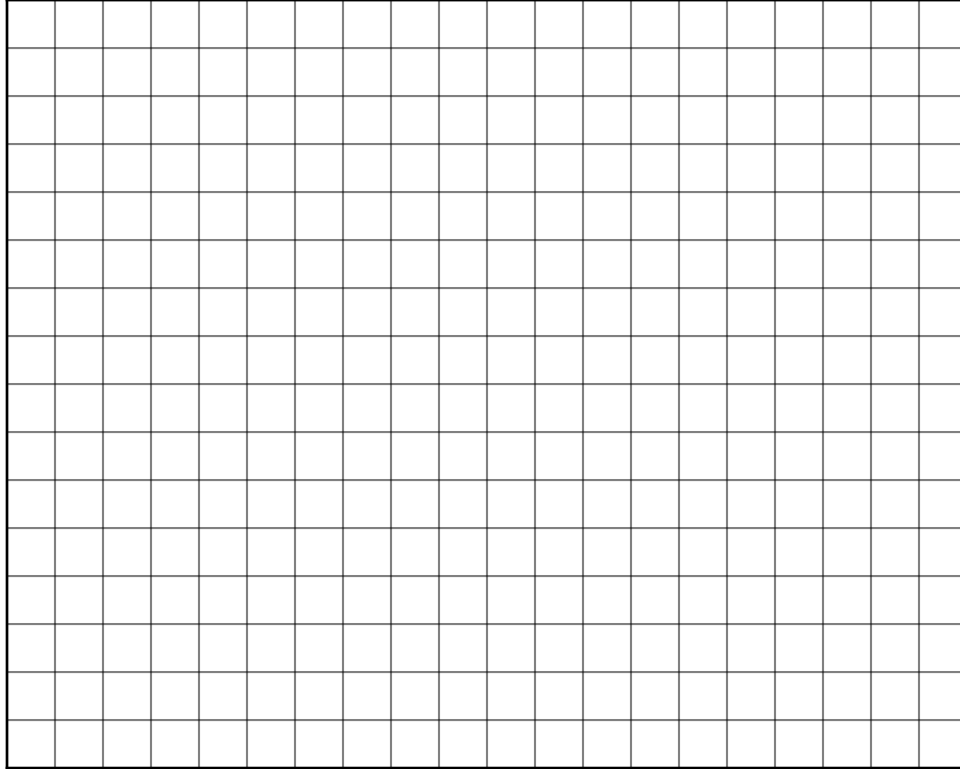
Example 14.7 (Activity: plot coffee price). Construct a plot showing the data from the table in Example 13.5.

Before you start drawing the plot, think carefully about plotting scale — what are the smallest and largest values you will need for both vertical and horizontal parts of the plot?



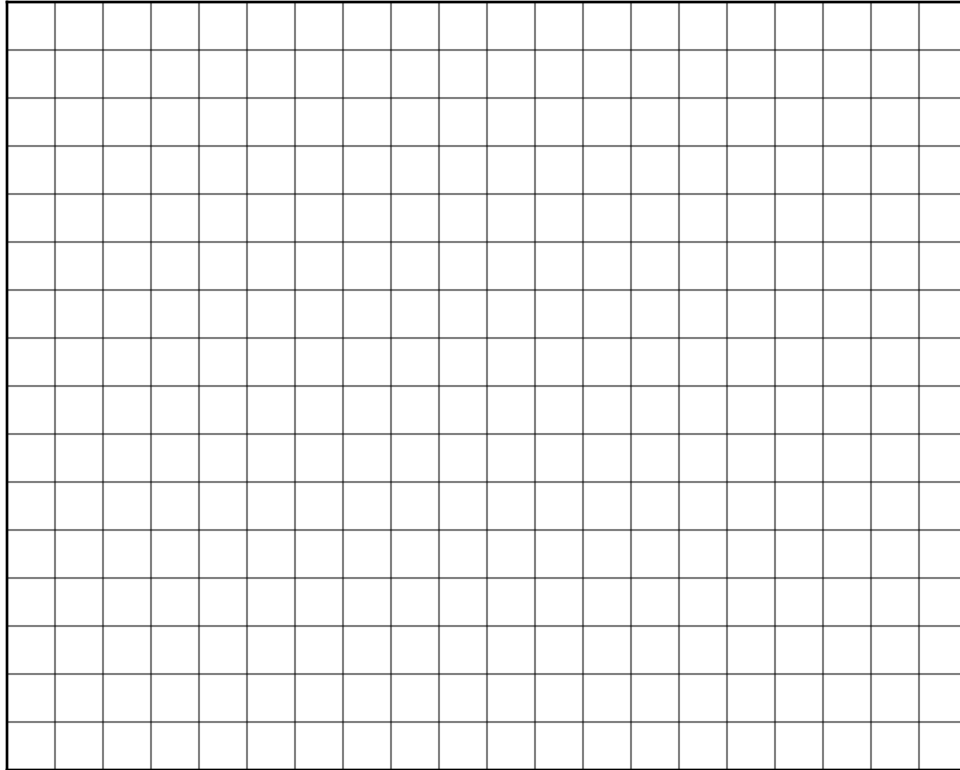
14.4. Homework exercises

Exercise 14.1. Construct a plot corresponding to the data table you made in Exercise 13.1.

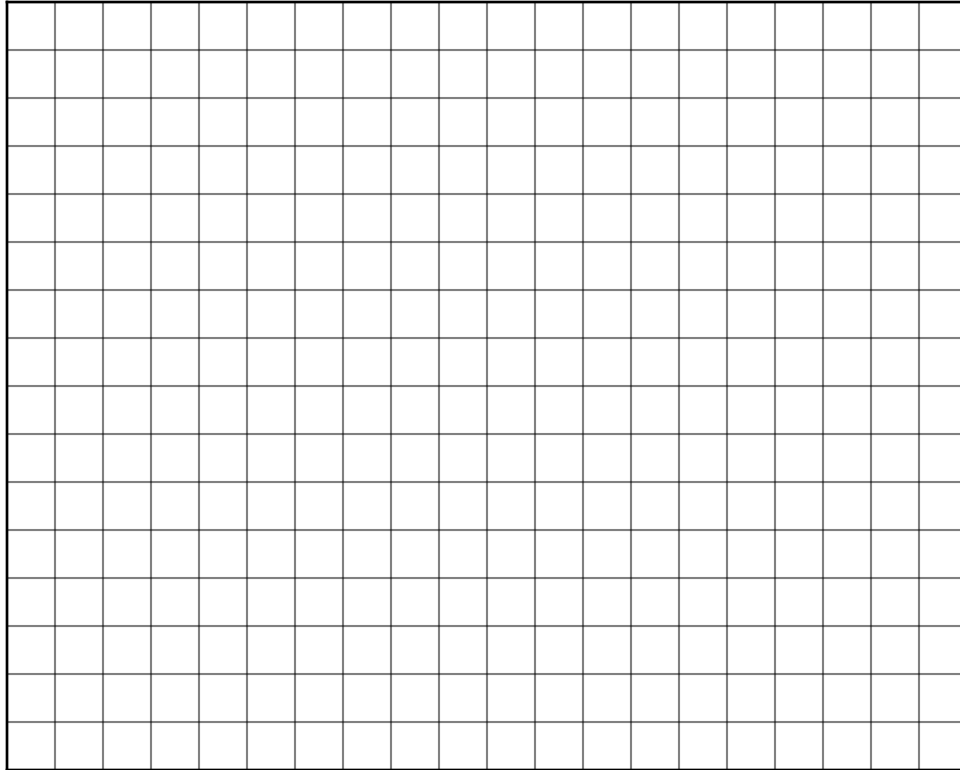


14. *Data plots*

Exercise 14.2. Construct a plot corresponding to the data table you made in Exercise 13.2.

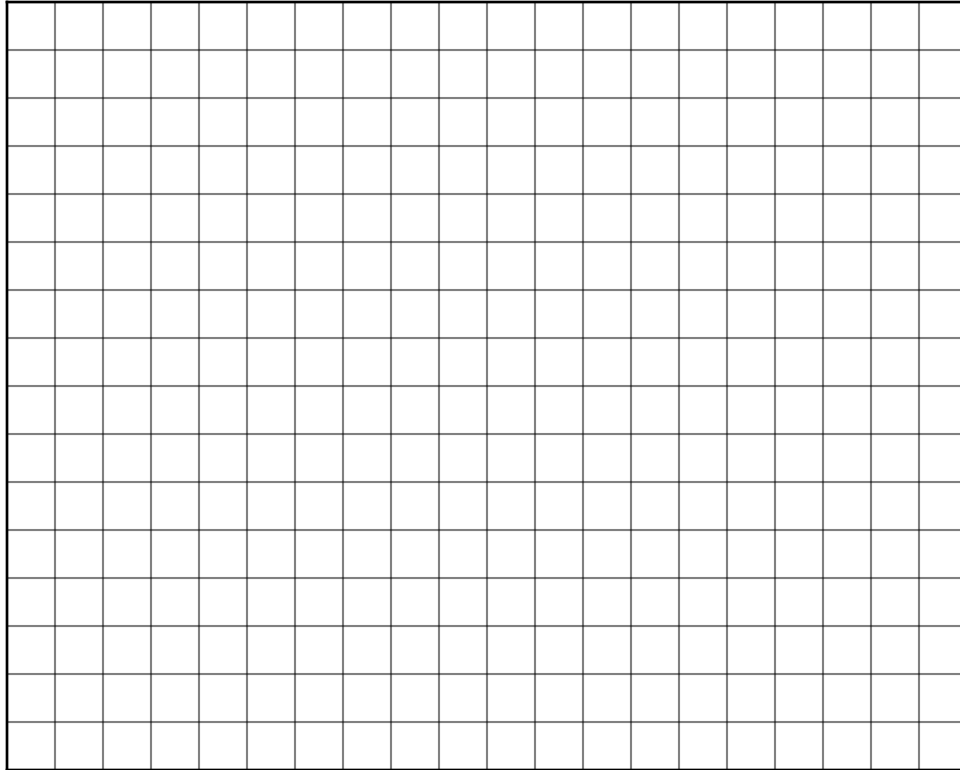


Exercise 14.3. Construct a plot corresponding to the data table you made in Exercise 13.3.

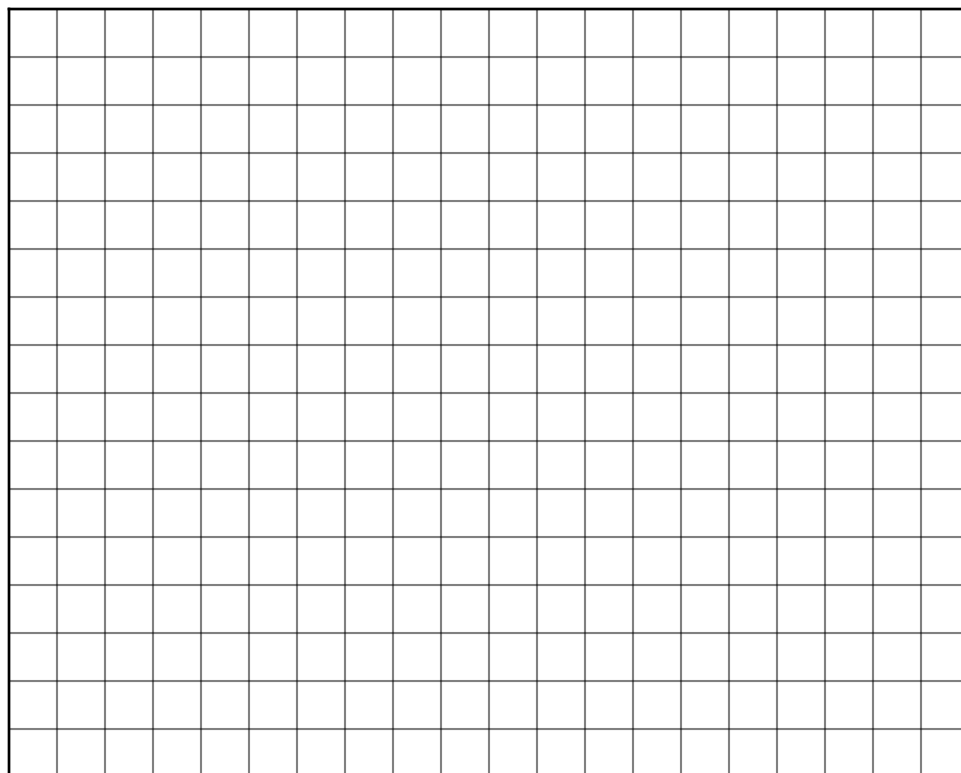


14. *Data plots*

Exercise 14.4. Construct a plot corresponding to the data table you made in Exercise 13.4.



Exercise 14.5. Construct a plot corresponding to the data table you made in Exercise 13.5.



15. Practice with data tables and plots

In this section, we have two goals:

- First, we want to be more systematic about setting the horizontal and vertical scales for our plots.
- Second, we practice using data tables and plots to describe various scenarios.

15.1. Choosing horizontal and vertical scales

Most of the graph paper pieces we have been using are 20 squares wide and 16 squares tall. How can we choose the horizontal and vertical scale to best use this space?

Example 15.1 (Activity: choosing plot scales). Sugar Paws, a dog who lives in El Prado, starts 5 meters away from her owners. She starts running away from her owners at 2 meters per second. We use the variables

- x is the number of seconds
- y is the distance from dog to owners

to describe the situation over the course of 10 seconds.

Table of Values

Create a table of values for this scenario.

$x = \text{time (seconds)}$	$y = \text{distance (meters)}$
0	
1	
2	
3	
:	
10	

15. Practice with data tables and plots

Horizontal Scale

We want x to range from ____ to ____.

In order to use as close to 20 boxes as best as possible, we choose a scale so that

$$\text{____ second(s)} = \text{____ box(es)}.$$

Vertical Scale

We want y to range from ____ to ____.

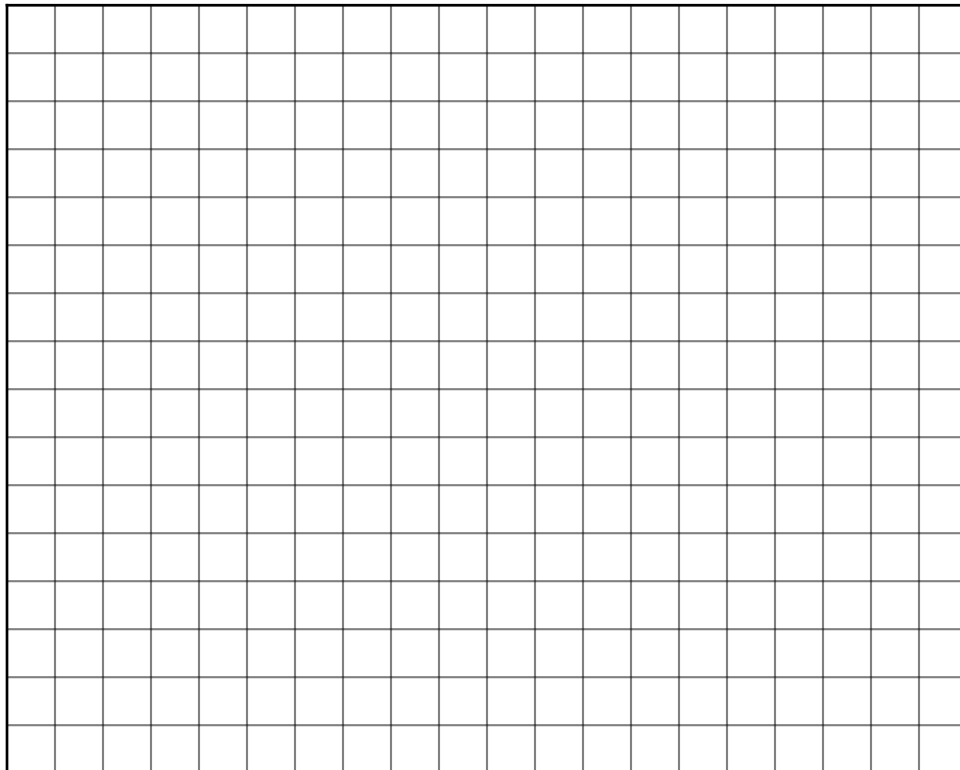
For convenience, we want to show $y = 0$ on the plot.

In order to use as close to 16 boxes as best as possible, we choose a scale so that

$$\text{____ meter(s)} = \text{____ box(es)}.$$

Plot

Use the scales you determined above to create a plot for this scenario. Fill in additional data points as well.



Example 15.2 (Activity: raffle gift card). Amie wins the UNM Faculty-Staff raffle, earning a \$100 gift card to a fancy chocolate shop near the plaza. Each week, she spends \$15 on a fancy mocha and croissant. Describe this scenario using the variables

- x is the number of weeks
- y is the balance on her gift card

to describe the scenario for 5 weeks.

Table of Values

Create a table of values for this scenario.

Horizontal Scale We want x to range from ____ to ____.

In order to use as close to 20 boxes as best as possible, we choose a scale so that

$$\text{____ week(s)} = \text{____ box(es)}.$$

Vertical Scale

We want y to range from ____ to ____.

For convenience, we want to show $y = 0$ on the plot.

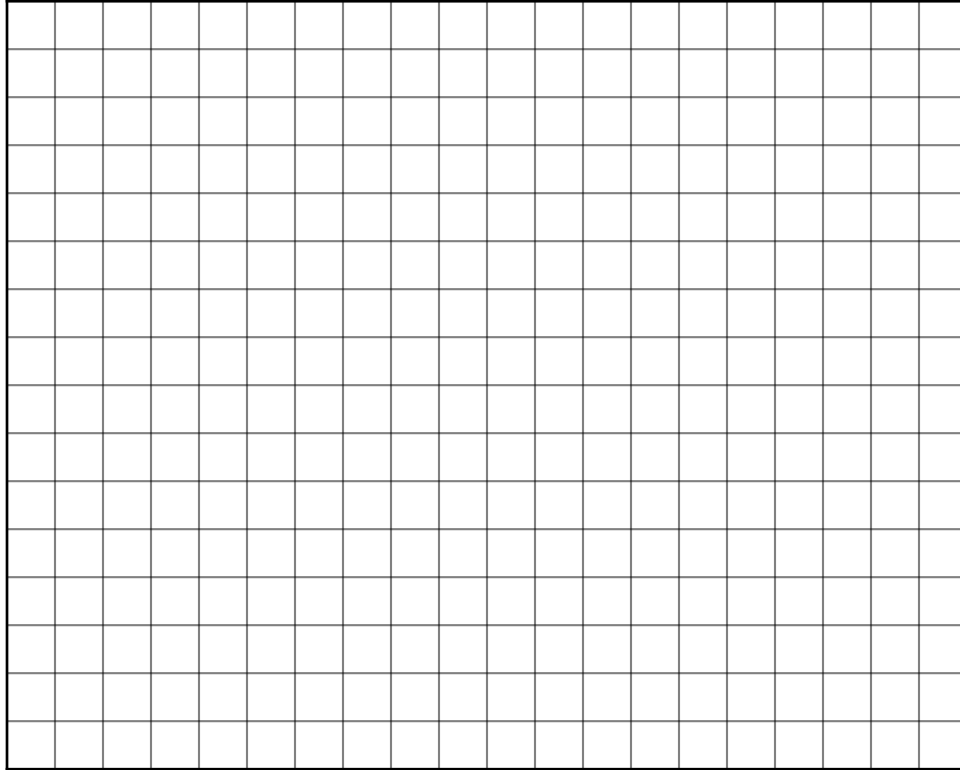
In order to use as close to 16 boxes as best as possible, we choose a scale so that

$$\text{____ dollar(s)} = \text{____ box(es)}.$$

15. Practice with data tables and plots

Plot

Use the scale you have chosen above to create a plot for this scenario.



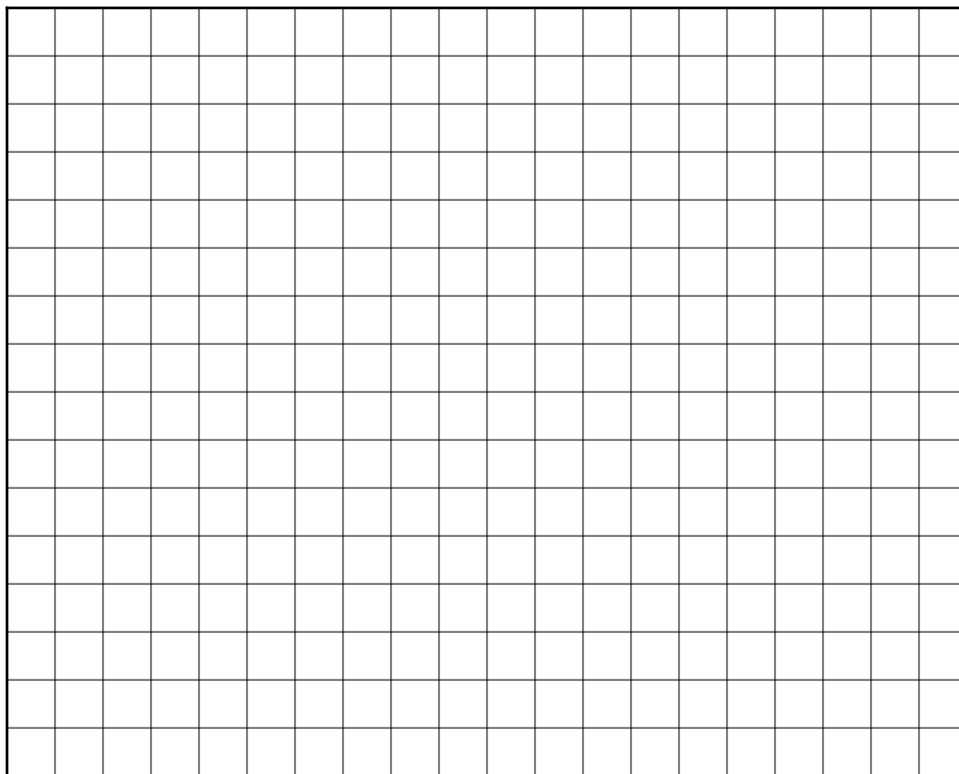
15.2. More modeling scenarios

We now consider more modeling scenarios. For each scenario below, complete the following tasks.

- Identify the key quantities of the scenario, and assign variables to each.
- Make a table of values showing pairs of related values.
- Construct a plot, based on your table of values.

Example 15.3 (Activity: driving to Denver). Mary is driving from Taos to Denver, averaging a speed of 60 miles per hour. The total distance for the trip is 360 miles.

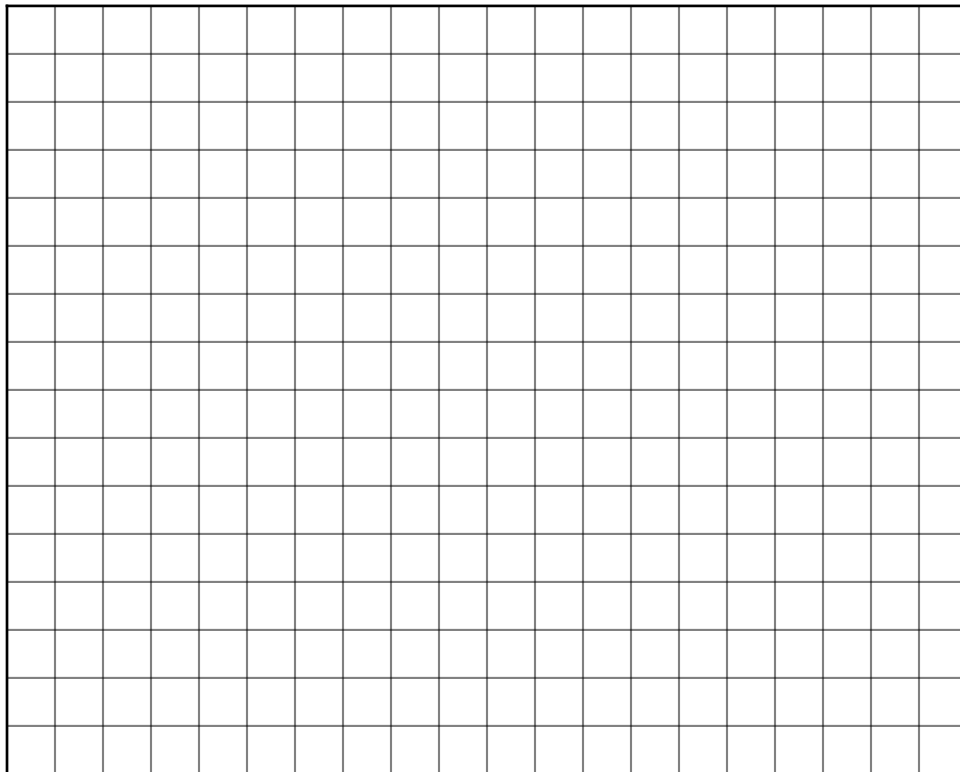
Your task is to describe how the time that Mary has driven relates to the distance remaining. (For example, after 1 hour, Mary still has 300 miles left to travel.)



15. Practice with data tables and plots

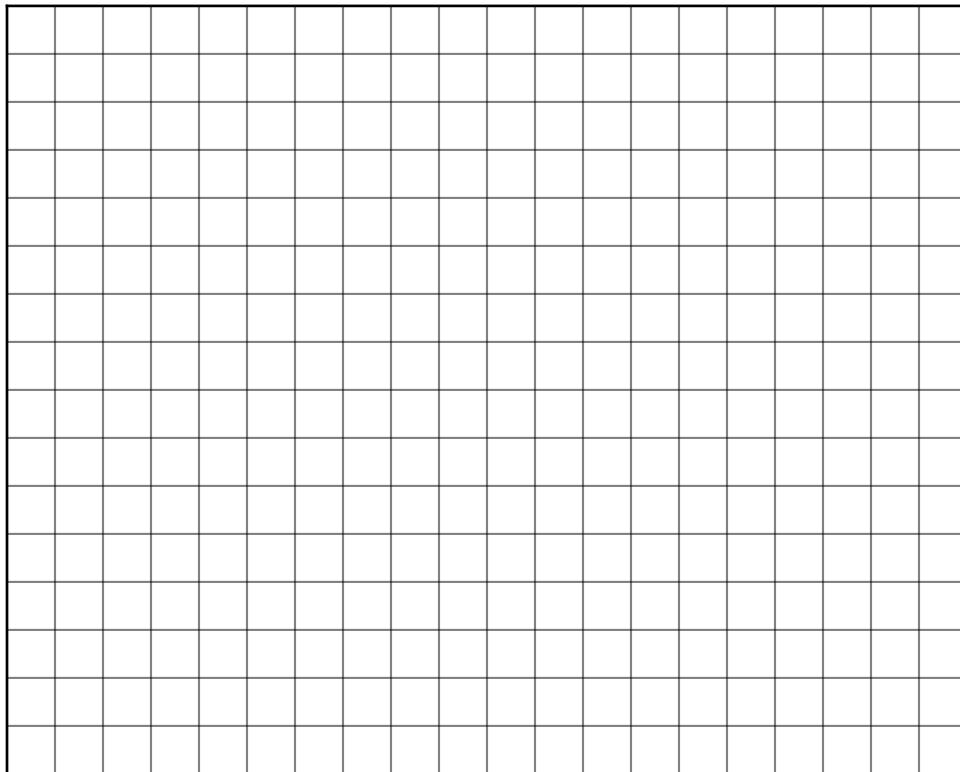
Example 15.4 (Activity: filling the tank). At *Fast Route Discount Gas*, unleaded fuel is currently priced at \$2.85 per gallon. Randi is filling her tank, which holds a total of 15 gallons.

Your task is to relate the amount of gas Randi puts in her car to the cost of the gas. (For example, two gallons of gas costs \$5.70.)



Example 15.5 (Activity: coffee card). Mandy is given a gift card to the Taos Bean Company for her birthday. The card comes with \$50 on it. Each day, she stops by and spends \$3.75 on a coffee drink.

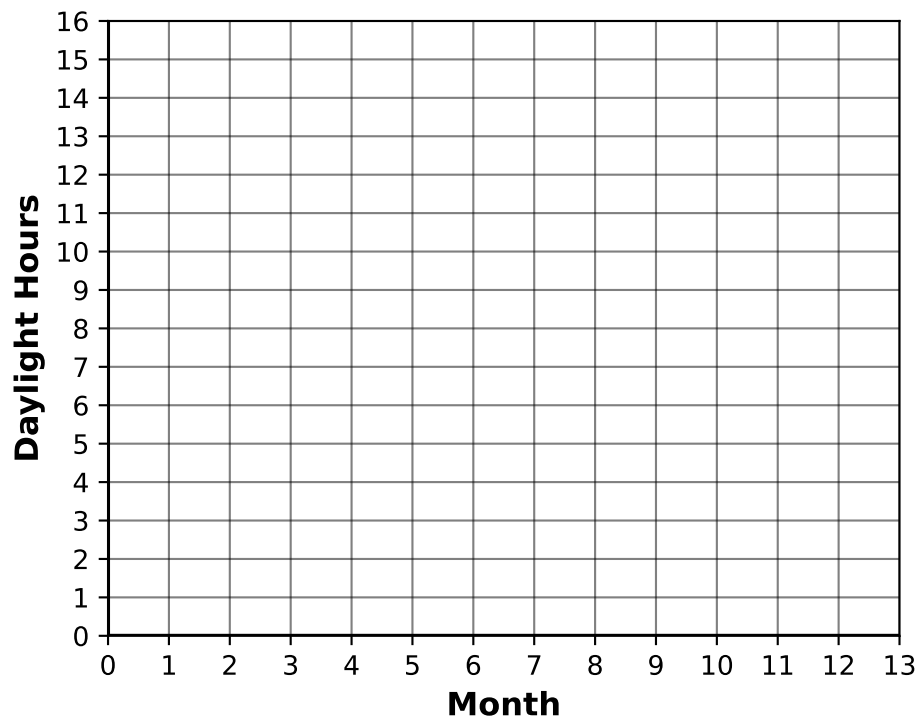
Your task is to relate the number of days that has gone by to the balance left on her card. (For example, after 1 day, the balance is \$46.25.)



15.3. Homework exercises

Exercise 15.1. The following table shows daylight hours in Taos, NM in terms of the month. Plot the data on the Cartesian plane below.

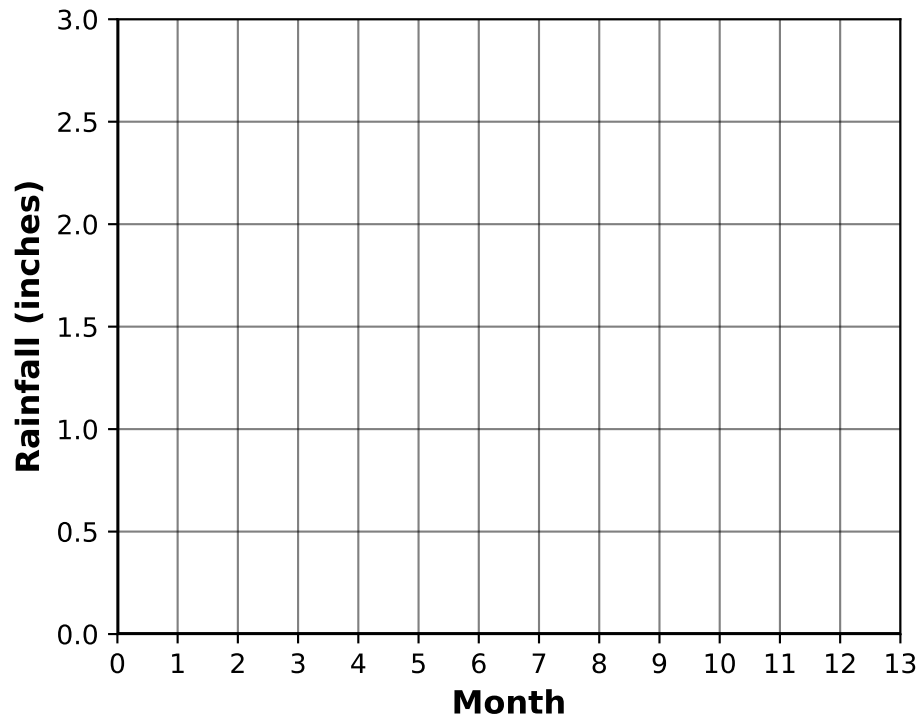
Month	Daylight Hours
1	9.5
2	10.5
3	12.0
4	13.5
5	14.5
6	15.0
7	14.5
8	13.5
9	12.5
10	11.0
11	9.5
12	9.0



Exercise 15.2. The following table shows average monthly rainfall in Taos, NM.

Month	Rainfall (inches)
1	0.6
2	0.7
3	0.9
4	1.0
5	1.2
6	1.3
7	2.4
8	2.6
9	1.8
10	1.3
11	0.8
12	0.7

Plot the data from the table in the coordinate grid below.

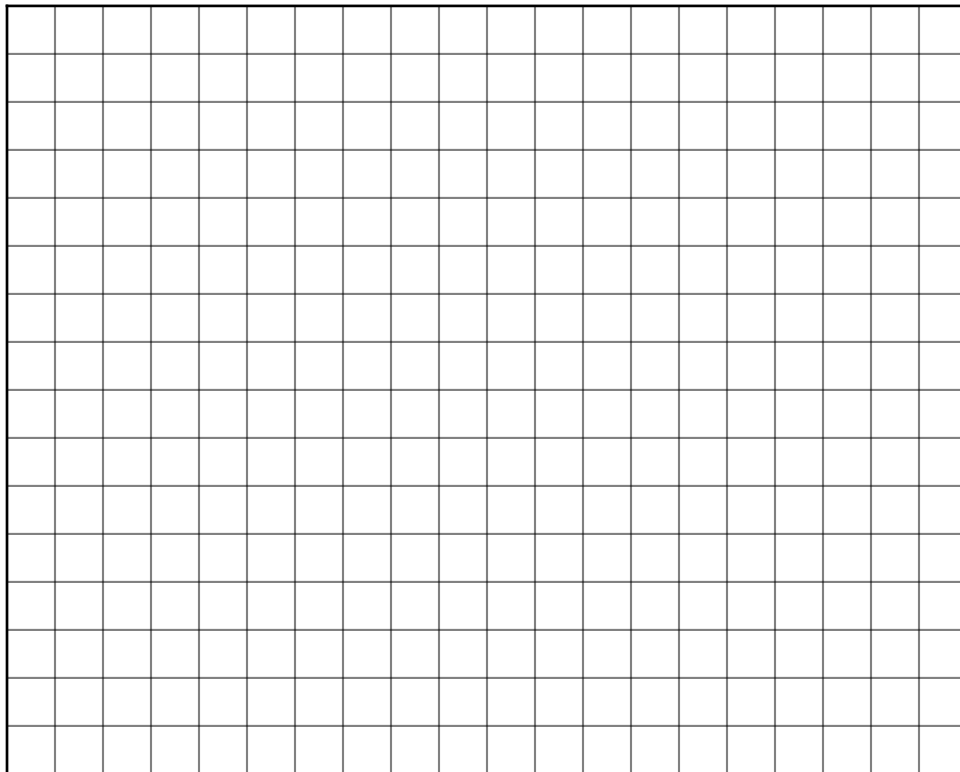


15. Practice with data tables and plots

Exercise 15.3. Paco has a side hustle selling cookies at the Taos Farmers Market. He charges \$6 for a bag of three cookies.

1. Make a table of values that relates the number of cookies sold (x) and the revenue that Paco generates (y). Your table should include the possibility that Paco can sell 10 bags of cookies in a single day.

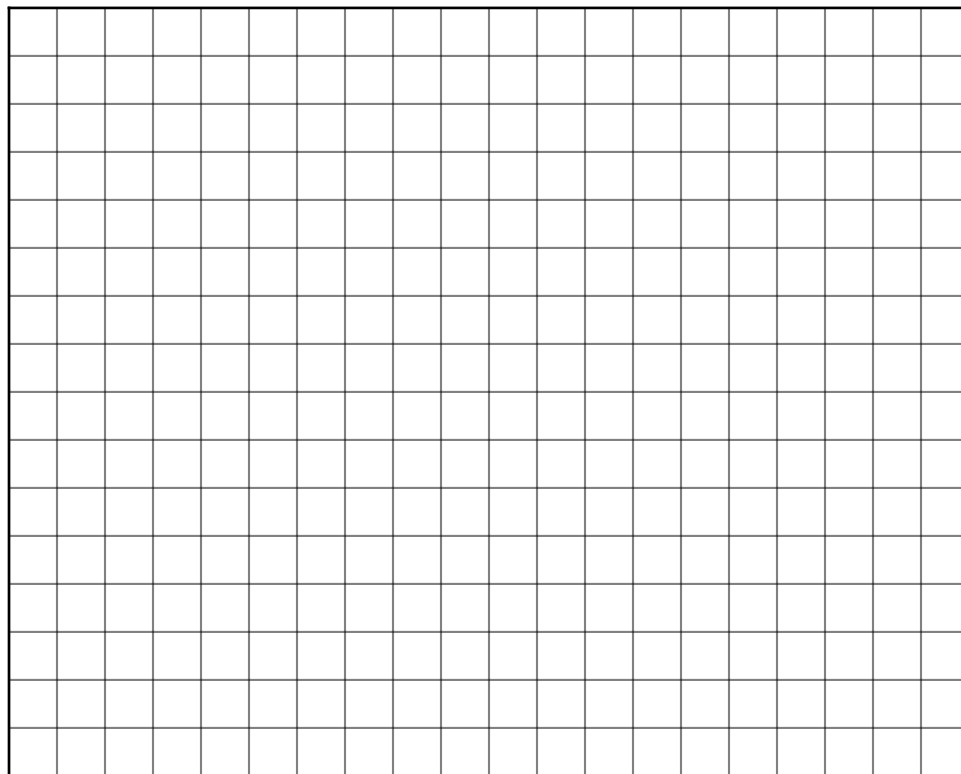
2. Make a plot of the data points from your table and connect the points with a line.



Exercise 15.4. The Taos Farmer's Market charges Paco for the privilege of selling at the market. They charge him a \$20 entry fee, and also charge him \$2 for every bag of cookies that he sells.

1. Make a table of values that relates the number of cookies he sells (x) to the fee that Paco is charged (y).

2. Make a plot of your data points and then construct a line.



Part V.
Linear Relations I

16. Linear vs nonlinear relations

In our previous exploration of data tables and plots, there were many examples where the plotted points were along a straight line.

Take a moment right now and look over the previous sections: which scenarios had data points in a straight line?

Our task in this section is to start to systematically identify what types of scenarios have straight line plots.

To address this issue, we consider four different scenarios. For each scenario, your task is to create a table of values and then to create the plot. Two of the scenarios will give us straight line plots, and two of the scenarios will not give us straight line plots. After we have analyzed the scenarios, we will discuss a general rule for determining what type of scenarios can be described with straight line plots.

16. *Linear vs nonlinear relations*

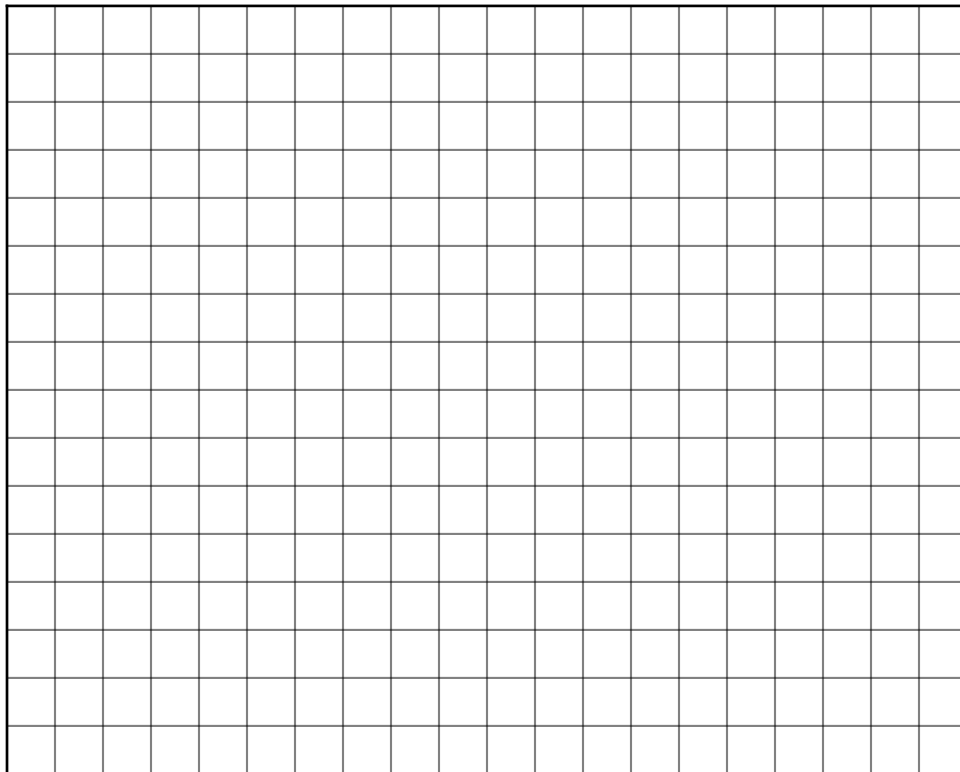
Example 16.1 (Activity: first chocolate scenario). A bowl of chocolates sits on the table in the Stem Learning Center. At the beginning of the week, there are 32 chocolates. Each day Paul takes half of the chocolates that are in the bowl.

Use the variables

- x is the number of days
- y is the number of chocolates left in the bowl.

First, make a table of values for this scenario.

Second, use your table of values to construct a plot for this scenario.



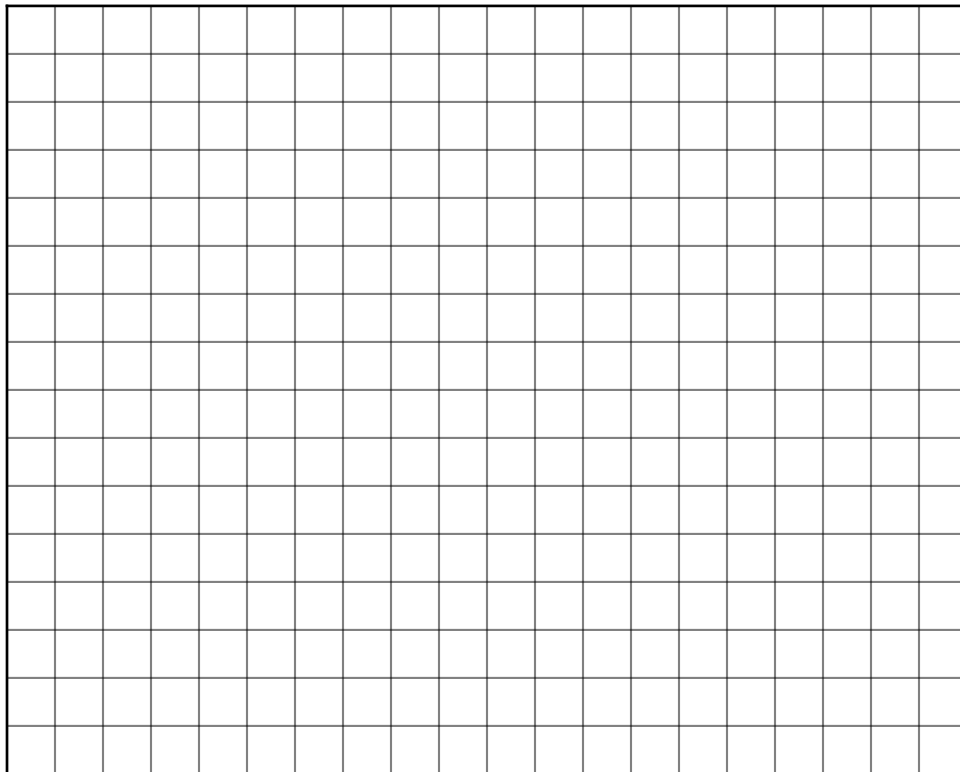
Example 16.2 (Activity: second chocolate scenario). A bowl of chocolates sits on the table in the Stem Learning Center. At the beginning of the week, there are 32 chocolates. Each day Paul takes 4 of the chocolates that are in the bowl.

Use the variables

- x is the number of days
- y is the number of chocolates left in the bowl

First, make a table of values for this scenario.

Second, use your table of values to construct a plot for this scenario.



16. *Linear vs nonlinear relations*

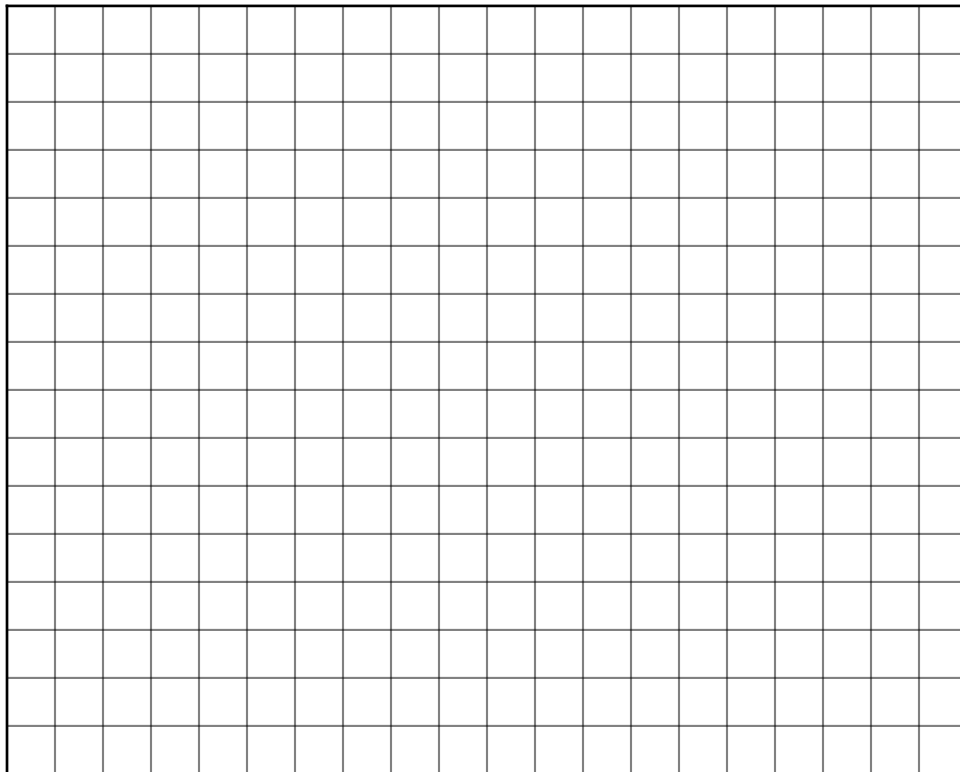
Example 16.3 (Activity: first woodchopping scenario). Pamona is chopping and stacking wood for the winter. At the beginning of the week, there are 10 pieces of wood in her woodpile. Each day she adds 20 pieces of wood to her woodpile.

Use the variables

- x is the number of days
- y is the number of pieces of wood in the woodpile

First, make a table of values for this scenario.

Second, use your table of values to construct a plot for this scenario.



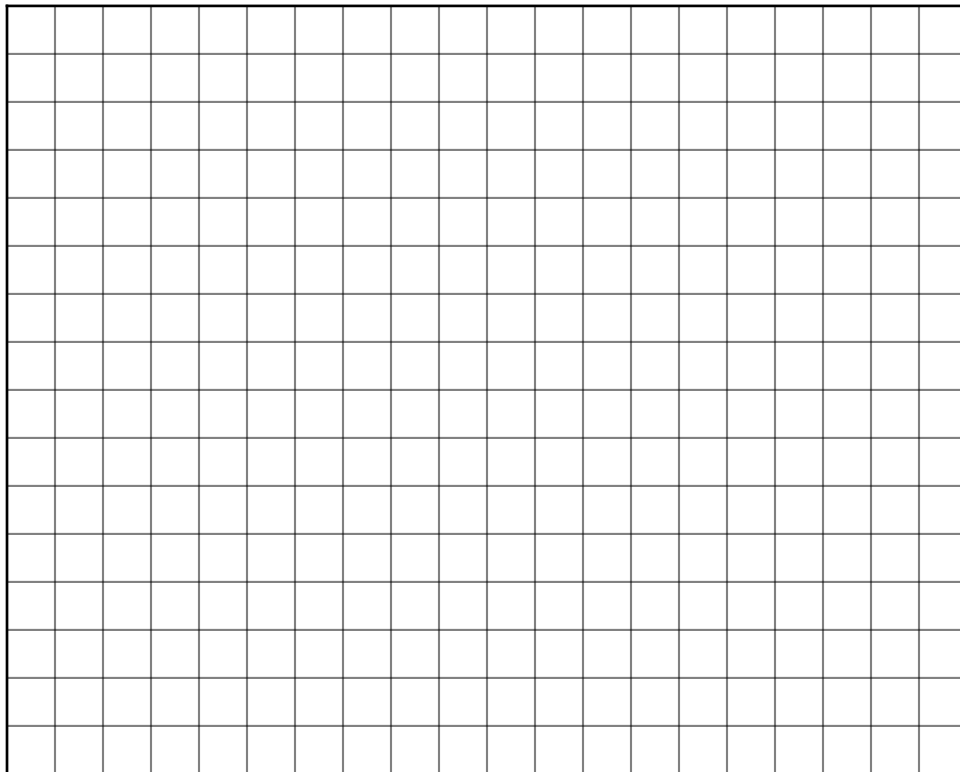
Example 16.4 (Activity: second woodchopping scenario). Pamona is chopping and stacking wood for the winter. At the beginning of the week, there are 10 pieces of wood in her woodpile. Each day she doubles the number of pieces of wood in her woodpile.

Use the variables

- x is the number of days
- y is the number of pieces of wood in the woodpile

First, make a table of values for this scenario.

Second, use your table of values to construct a plot for this scenario.



16. *Linear vs nonlinear relations*

16.1. Group discussion

Which scenarios have straight line plots? Which did not have straight line plots?

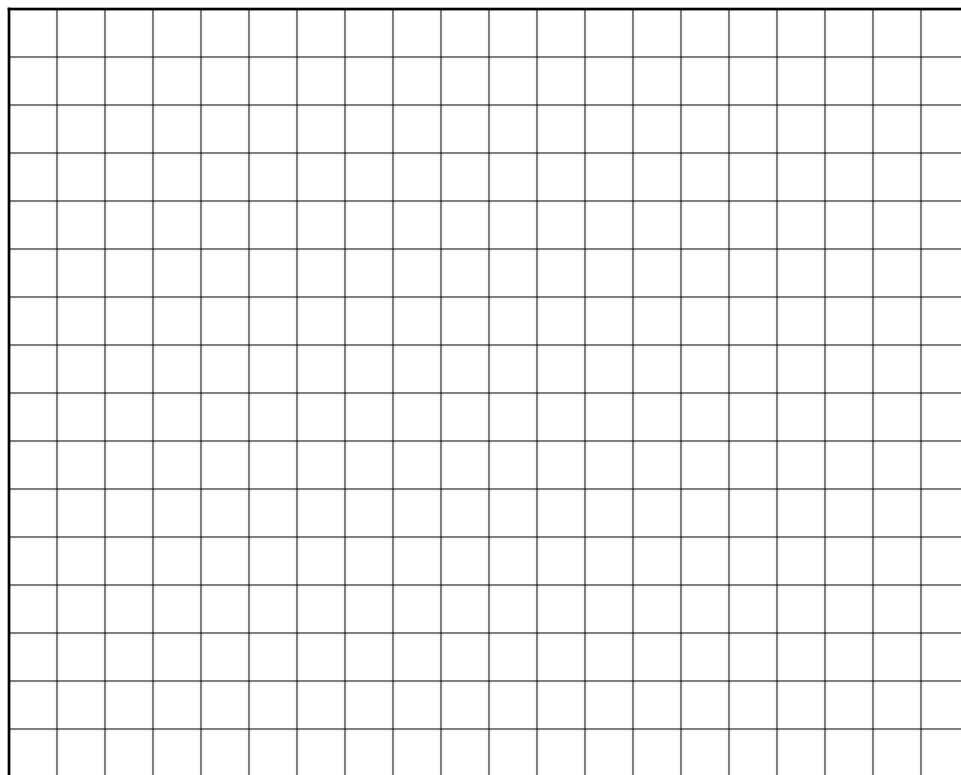
What are the features of scenarios that will have straight line plots?

16.2. Homework exercises

Exercise 16.1. Consider the following table of values. Construct a plot, and then determine whether or not the data is linear.

After you have made the determination, revisit the table. Would it have been possible to deduce that the data is linear by only using the table (and not relying on the plot)?

x	y
0	5
1	8
2	11
3	14
4	17
5	20

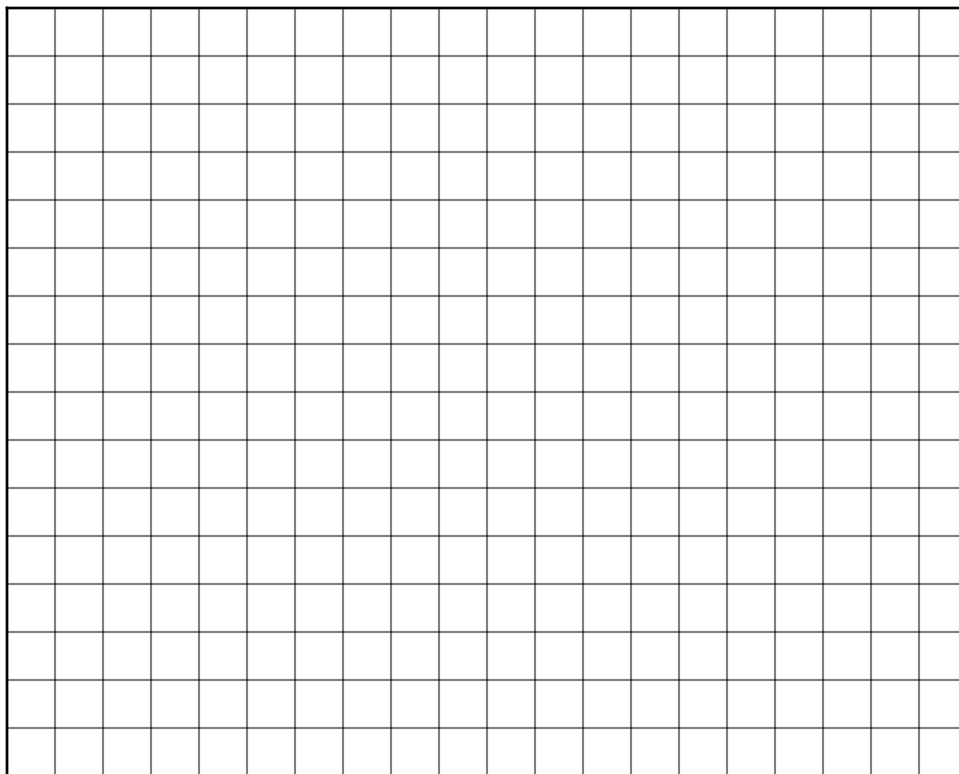


16. *Linear vs nonlinear relations*

Exercise 16.2. Consider the following table of values. Construct a plot, and then determine whether or not the data is linear.

After you have made the determination, revisit the table. Would it have been possible to deduce that the data is linear by only using the table (and not relying on the plot)?

<u>x</u>	<u>y</u>
0	1
1	2
2	5
3	10
4	17
5	26



Exercise 16.3. The following is a list of scenarios. Try to determine, without plotting, whether the scenario will be linear or not.

1. Father Mike lights a candle at the church. Originally, the candle is 10 inches tall. Each hour, the candle burns away $\frac{1}{2}$ inch of height. Does the height of the candle change linearly?
2. Mayor Dan is serving pancakes at the Plaza. He starts with a supply of 400 pancakes. During the first hour, half of the pancakes are eaten. During the second hour, half of the remaining pancakes are eaten. Does the number of remaining pancakes change linearly?
3. Colin is making a collection of wooden boxes to hold telescope lenses. The boxes are square, but with different widths. Does the area of the box change linearly when the width is changed?
4. Colin is making a collection of wooden boxes to hold telescope lenses. The boxes are square, but with different widths. Does the perimeter of the box change linearly when the width is changed?

16. *Linear vs nonlinear relations*

Exercise 16.4. Construct two tables of values as follows:

1. Construct a data table where the plot will be linear. The table needs to have five rows in it.
2. Construct a data table where the plot will *not* be linear. The table needs to have five rows in it.

After you have constructed your tables, make plots for each.

Exercise 16.5. Construct two scenarios as follows:

1. Construct a scenario where the corresponding data plot will be linear. Make both a data table and a plot for your scenario.
2. Construct a scenario where the corresponding data plot will *not* be linear. Make both a data table and a plot for your scenario.

17. The linear model framework

A *linear model* is a mathematical description of a scenario in which the plotted data all lies on a straight line. Our goal for this section is to develop a systematic approach to constructing and interpreting linear models.

17.1. Generic linear model

If a scenario can be described in terms of a *starting value* and *constant rate*, then a linear model is appropriate.

- The starting value is also called the *base value* and is usually given the letter b .
- The *rate* is usually given the letter m .

We usually use the variable x to represent the input to our model, which tells us the number of rate steps. And we use the letter y to represent the output of our model, which is the amount.

In words, our model can be described by

$$\text{Resulting Amount} = (\text{Base Value}) + (\text{Rate})(\text{Number of Rate Steps}).$$

In terms of the letters introduced above, the model is

$$y = b + mx.$$

i Units for rate and base

It can be helpful to use units to determine what is the rate and what is the base value.

The base value has the same units as the amount y that we are modeling.

The rate has units of ___ per ___, where the first blank is the units of y and the second blank is the units of x .

17. The linear model framework

Example 17.1. In Example 15.1 we considered the scenario of the dog Sugar Paws running away from her owner.

- Sugar Paws started 5 meters away from her owner. This is the base value, and so we have $b = 5$.
- Sugar Paws is running at 2 meters per second. This is the rate, and so we have $m = 2$.

Using our previously defined variables

- x is the number of seconds
- y is the distance from dog to owners

we can form our linear model

$$y = 5 + 2x.$$

i Equation for a linear model

Equations that can be written in the format

$$y = b + mx,$$

where b and m are fixed numbers representing base and rate, are called *linear equations* or *linear models*.

Example 17.2 (Activity: Amie's raffle gift card revisited). In the previous section, we also considered the situation where Amie had won the UNM Faculty-Staff raffle, earning a \$100 gift card. She then went on to spend \$15 from the card each week.

In our model, we are describing $y =$ the amount left on the gift card in terms of $x =$ the number of weeks Amie has had the card.

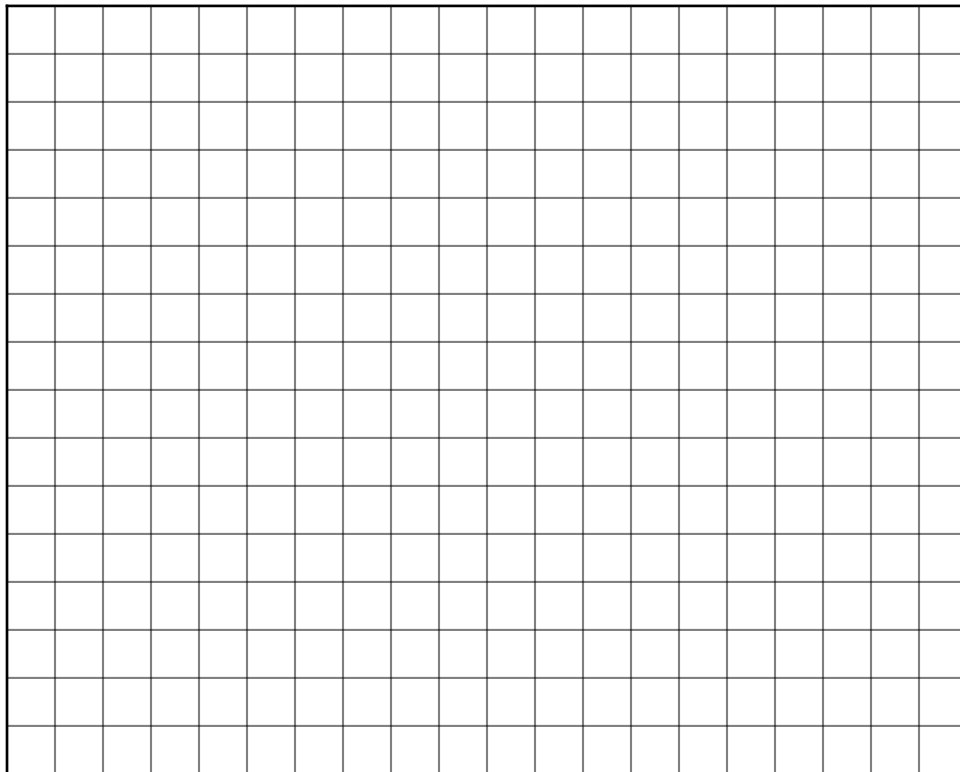
- What is the base value in this scenario?

- What is the rate in this scenario?

Use this information to construct the linear equation that describes this scenario.

Example 17.3 (Activity: Paul's Taco Emporium). At Paul's Taco Emporium, customers are charged \$5 for a basic plate of rice and beans, plus an additional \$2 per taco.

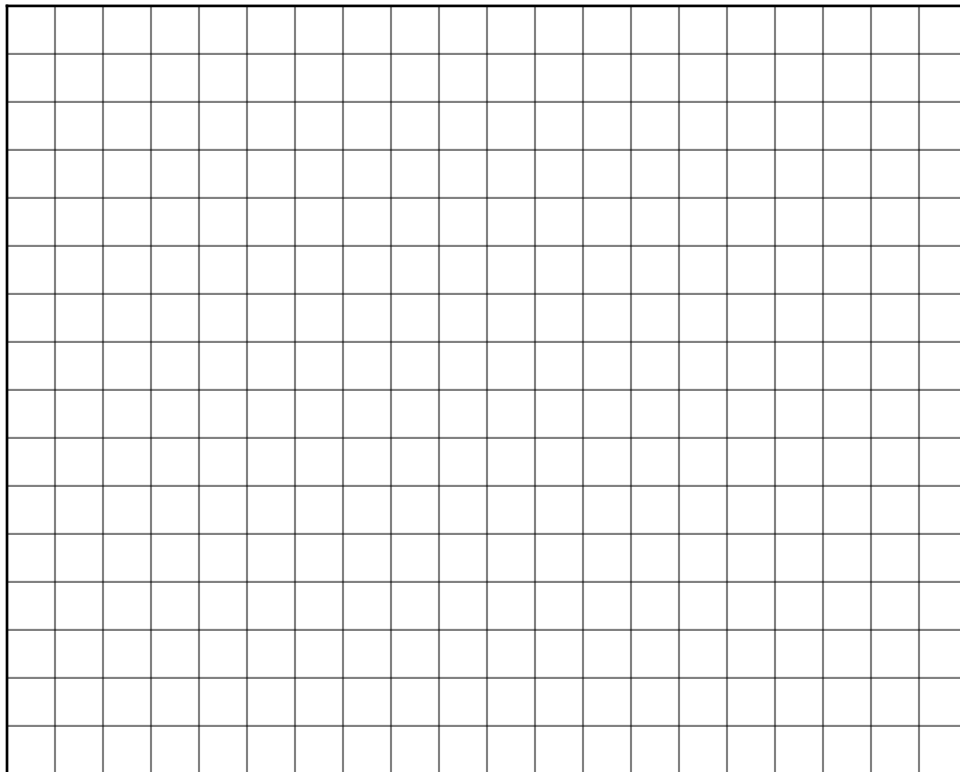
- What is the base value? What is the rate?
- Make a table of values where x is the number of tacos and where y is the total cost of the order.
- Make a plot that shows the values in your table.
- Make an equation that gives cost in terms of the number of tacos.
- How many tacos can we get if we are willing to spend \$23? Show your work!



17. *The linear model framework*

Example 17.4 (Activity: Lucas' Taco Empire). At Lucas' Taco Empire, customers are charged \$3 for a basic plate of rice and beans, plus an additional \$4 per taco.

- What is the base value? What is the rate?
- Make a table of values where x is the number of tacos and where y is the total cost of the order.
- Make a plot that shows the values in your table.
- Make an equation that gives cost in terms of the number of tacos.
- How many tacos can we get if we are willing to spend \$23? Show your work!



17.2. Homework exercises

Exercise 17.1. After a summer monsoon rain shower, a cistern in Taos has been filled to 250 gallons of water. Each day, 25 gallons are used to water the garden.

Let x = the number of days since the rain shower, and let y = the amount of water remaining in the cistern (in gallons).

What is the base value in this scenario? What units does it have? What does it represent?

What is the rate in this scenario? What units does it have? What does it represent?

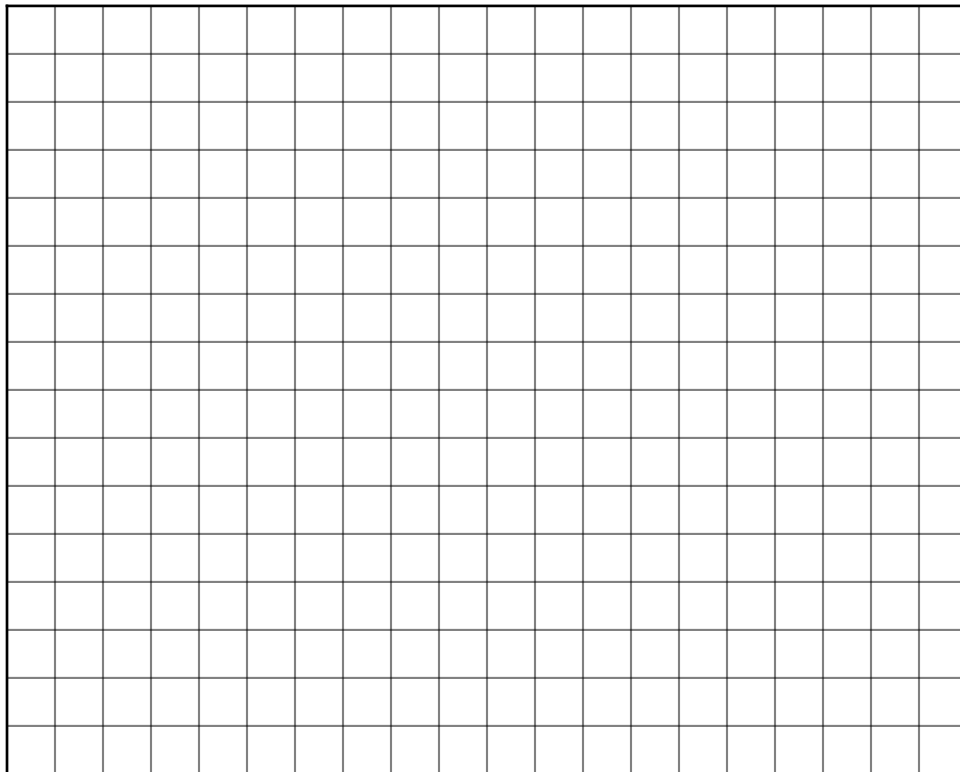
Write the linear equation that describes the amount of water remaining in the cistern.

After 4 days, how much water is left in the cistern? Show your work.

After how many days will exactly 50 gallons remain in the cistern? Show your work.

17. *The linear model framework*

Make a table of values for this model, and draw the plot below.



Exercise 17.2. At the start of the semester, there are 30 random papers piled in the corner of Professor Paul's desk. Each week, an additional 20 papers accumulate on the pile.

Let x = the number of weeks since the start of the semester, and let y = the number of papers on the desk.

What is the base value in this scenario? What units does it have? What does it represent?

What is the rate in this scenario? What units does it have? What does it represent?

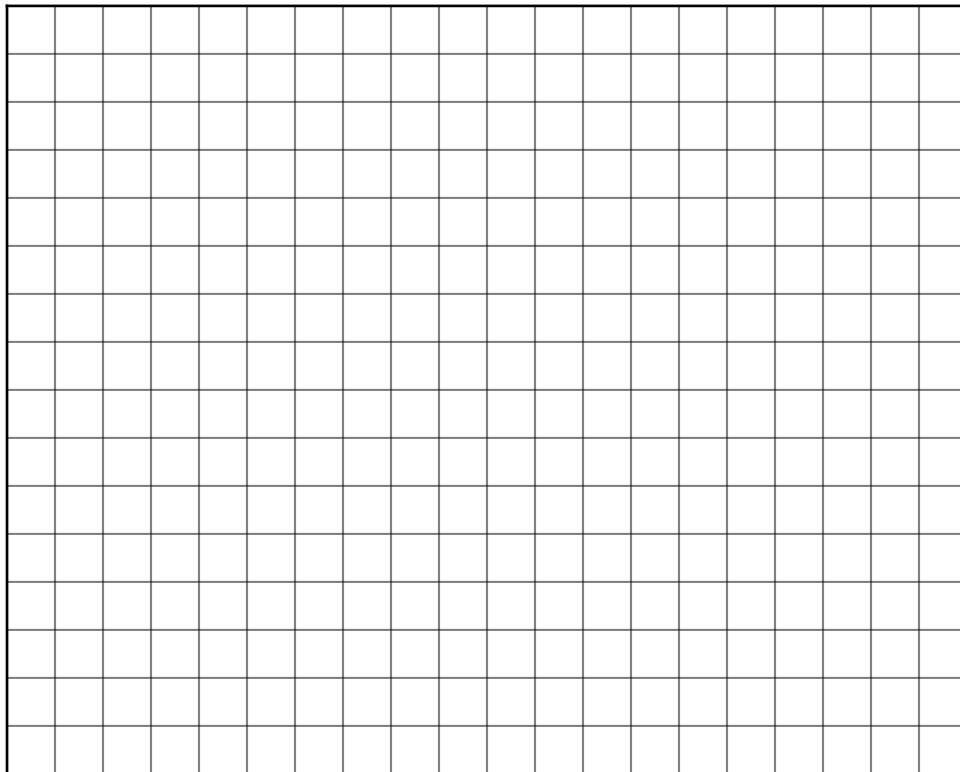
Write the linear equation that describes the number of papers on the desk.

How many papers are on the desk after 4 weeks? Show your work.

After how many weeks will there be 150 papers on the desk? Show your work.

17. *The linear model framework*

Make a table of values for this model, and draw the plot below.



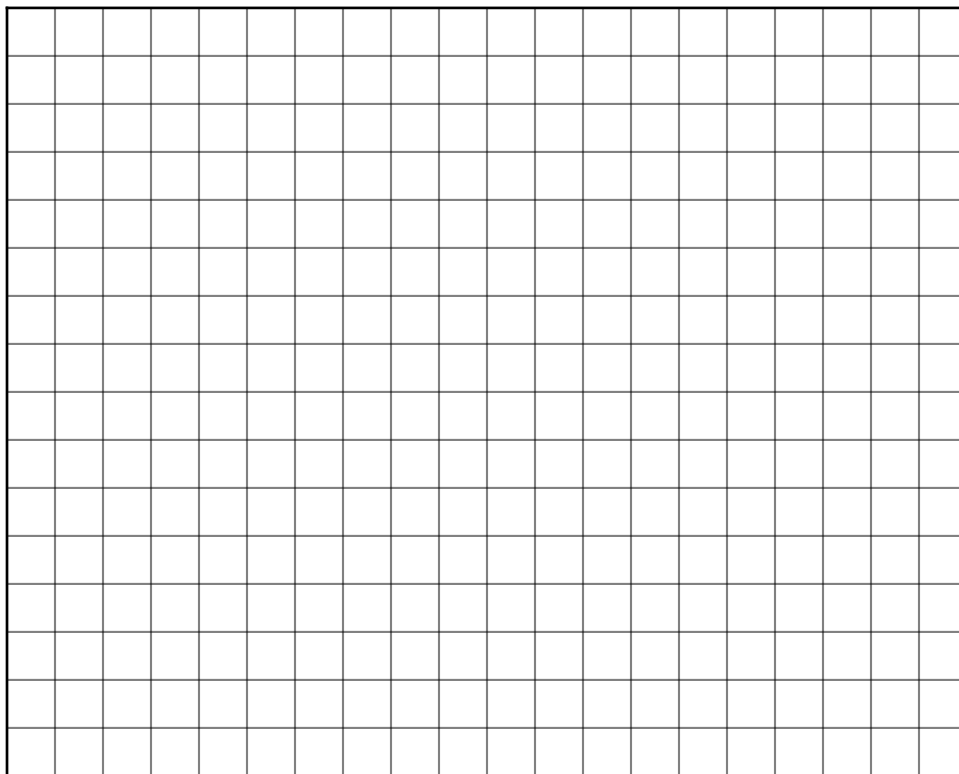
Exercise 17.3. Mark is working at a famous burrito cart on Paseo del Pueblo Sur. His daily earnings depend on how many burritos he sells. Using x to represent the number of burritos he sells and y to represent his daily earnings, we have the formula

$$y = 100 + 0.25x.$$

What number is the base in this scenario? What units does the base have? What does this number represent in the context of this scenario?

What number is the rate in this scenario? What units does the rate have? What does this number represent in the context of this scenario?

Make a table of values, and draw the plot, for this model.



17. *The linear model framework*

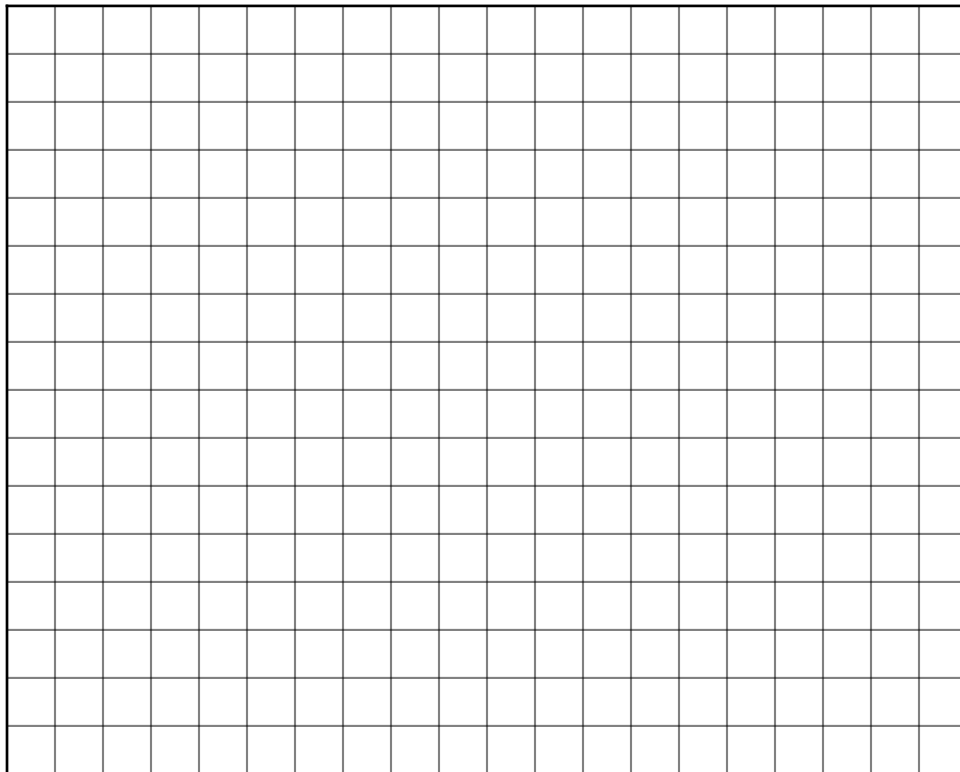
Exercise 17.4. Eric is working at a different, but also famous, burrito cart on Paseo del Pueblo Sur. His daily earnings depend on how many burritos he sells. Using x to represent the number of burritos he sells and y to represent his daily earnings, we have the formula

$$y = 80 + 0.50x.$$

What number is the base in this scenario? What units does the base have? What does this number represent in the context of this scenario?

What number is the rate in this scenario? What units does the rate have? What does this number represent in the context of this scenario?

Make a table of values, and draw the plot, for this model.



18. Linear model analysis

In this section we practice analyzing linear equations.

18.1. Interpreting linear formulas

In the previous scenarios we had to construct the formula. Now we consider the reverse problem: suppose we are given the formula and asked to interpret it.

Example 18.1 (Activity: Interpreting $y = 7 + 3x$). Consider the linear relation $y = 7 + 3x$.

Make a table of values.

Sketch the plot, without graph paper.

Suppose $y = 22$. What is x ? Show your work, and also indicate this on the graph.

Suppose $x = -3$. What is y ? Show your work, and also indicate this on the graph.

18. *Linear model analysis*

Example 18.2 (Activity: Interpreting $y = -3 + 4x$). Consider the linear relation $y = -3 + 4x$. Make a table of values.

Sketch the plot, without graph paper.

Suppose $y = 22$. What is x ? Show your work, and also indicate this on the graph.

Suppose $x = -3$. What is y ? Show your work, and also indicate this on the graph.

Example 18.3 (Activity: Interpreting $y = 6 - 2x$). Consider the linear relation $y = 6 - 2x$.
Make a table of values.

Sketch the plot, without graph paper.

Suppose $y = 22$. What is x ? Show your work, and also indicate this on the graph.

Suppose $x = -3$. What is y ? Show your work, and also indicate this on the graph.

18. *Linear model analysis*

Example 18.4 (Activity: Interpreting $y = 6 + x$). Consider the linear relation $y = 6 + x$. Make a table of values.

Sketch the plot, without graph paper.

Suppose $y = 22$. What is x ? Show your work, and also indicate this on the graph.

Suppose $x = -3$. What is y ? Show your work, and also indicate this on the graph.

18.2. Challenge: determining the rate from data

Sometimes the scenario is not described in a way that clearly identifies the base and the rate. The next activity explores this situation.

Example 18.5 (Activity: Jim's parking lot). Jim is running a private parking lot near the Alamosa Airport. He charges \$12 for 2 days. He charges \$18 for 4 days. We are going to assume that we have a linear relation between days and price.

- Make a graph of the data. Extend your graph to a straight line.
- Use your graph to figure out the base value for the linear relation.
- Use your graph to figure out the rate for the linear relation.
- What is the formula for cost in terms of number of days?
- How much does it cost to park at Jim's lot for 7 days?
- How long can you park at Jim's lot before the cost exceeds \$50?

18. *Linear model analysis*

18.3. Homework exercises

Exercise 18.1. Consider the linear model $y = 2x + 6$.

Make a small table of values.

Sketch the plot, indicating the key features.

Find x when $y = 16$. Show your work.

Find y when $x = -2$. Show your work.

Exercise 18.2. Consider the linear model $y = 3x - 2$.

Make a small table of values.

Sketch the plot, indicating the key features.

Find x when $y = 16$. Show your work.

Find y when $x = -2$. Show your work.

18. *Linear model analysis*

Exercise 18.3. Consider the linear model $y = 32 - 4x$.

Make a small table of values.

Sketch the plot, indicating the key features.

Find x when $y = 20$. Show your work.

Find y when $x = -2$. Show your work.

Exercise 18.4. Consider the linear model $y = 11 + x$.

Make a small table of values.

Sketch the plot, indicating the key features.

Find x when $y = 15$. Show your work.

Find y when $x = -2$. Show your work.

Exercise 18.5. Lucas is planning to open a competing parking lot at the Alamosa Airport. He will charge \$16 for 2 days and will charge \$24 for 6 days. We are going to assume that Lucas uses a linear relation between days and price.

- Make a graph of the data. Extend your graph to a straight line.
- Use your graph to figure out the base value for the linear relation.
- Use your graph to figure out the rate for the linear relation.
- What is the formula for cost in terms of number of days?
- How much does it cost to park at Lucas' lot for 7 days?
- How long can you park at Lucas' lot before the cost exceeds \$50?

Part VI.

Linear Relations II

19. The concept of slope

The goal for this section is to better understand the rate part of linear models. We want to gain a geometric understanding of the rate, as well as learn how to compute the rate from a data table. We begin with two motivating examples.

Example 19.1. Jim is running a private parking lot near the Alamosa Airport. He charges \$12 for 2 days. He charges \$18 for 4 days. We are going to assume that we have a linear relation between days and price.

1. First we the plot for this scenario, with time on the horizontal axis and price on the vertical axis. Put the two data points on the sketch.

2. What are the units of the rate?

3. Compute the rate, writing it as a fraction.

19. *The concept of slope*

Example 19.2. Martin is driving to Denver. After 2 hours, he has traveled 100 miles. After 4 hours, he has traveled 210 miles.

1. Sketch the plot for this scenario, with time on the horizontal axis and distance on the vertical axis. Put the two data points on the sketch.

2. What are the units of the rate?

3. Compute the rate, writing it as a fraction.

19.1. General formula for rate

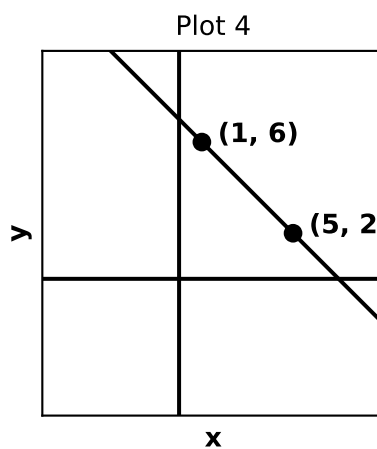
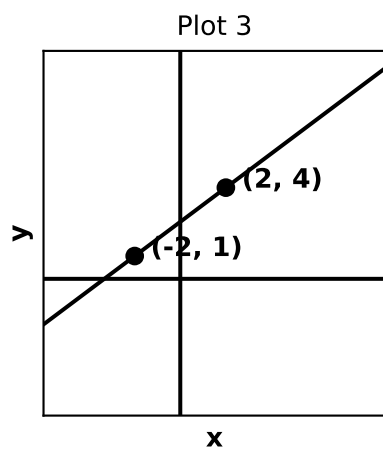
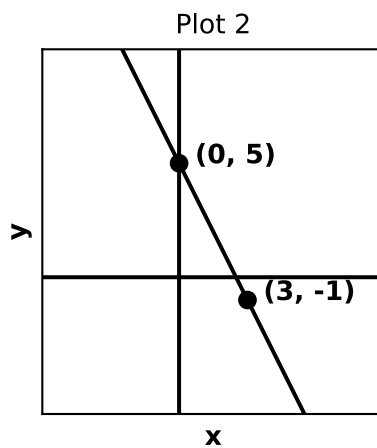
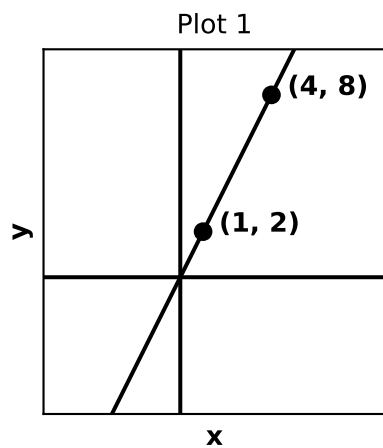
With x as the horizontal (input) variable and y as the vertical (output) variable, we can compute the rate as follows:

rate =

Sketch a generic plot and indicate the two parts of the fraction on the plot.

19.2. Computing the rate from the plot

Compute the rate for each of the following lines.



Discussion:

1. For which plots is the rate positive? For those plots, which rate is larger?
2. For which plots is the rate negative? For those plots, which rate is a larger negative value?
3. The rate is also called the *slope* of the line. Why does this word make sense?

19.3. Computing slope from a data table

Example 19.3 (Activity: Computing slope from a data table). Use the two points in the data table to compute the slope. Then sketch the line.

x	y
2	7
5	16

19. *The concept of slope*

Example 19.4 (Activity: Computing slope from a data table). Use the two points in the data table to compute the slope. Then sketch the line.

x	y
0	10
4	2

Example 19.5 (Activity: Computing slope from a data table). Use the two points in the data table to compute the slope. Then sketch the line.

x	y
-3	1
2	11

19. *The concept of slope*

Example 19.6 (Activity: Computing slope from a data table). Use the two points in the data table to compute the slope. Then sketch the line.

x	y
1	-2
6	-12

19.4. Discussion: We have the slope, what about the base?

In the previous examples, we were able to compute the slope from the two data points. How can we use the data to compute the value of the base b ?

Example 19.7 (Activity: Finding the equation from two points). Find the equation of the line having the points $(3, 5)$ and $(8, 20)$.

19. *The concept of slope*

Example 19.8 (Activity: Finding the equation from two points). Find the equation of the line having the points $(1, 13)$ and $(4, 5)$.

19.5. Section 19.3 revisited

Example 19.9 (Activity: Finding the base). In Example 19.3 you found the slope of the line passing through the data table. Now compute the value of the base b to get a complete linear equation.

19. *The concept of slope*

Example 19.10 (Activity: Finding the base). In Example 19.4 you found the slope of the line passing through the data table. Now compute the value of the base b to get a complete linear equation.

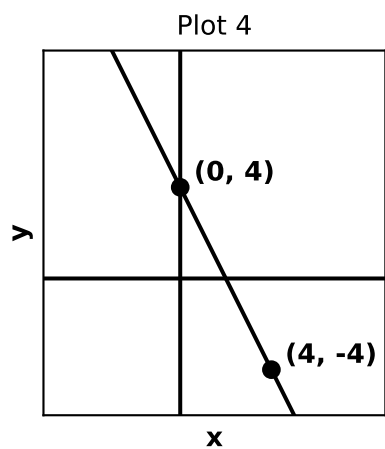
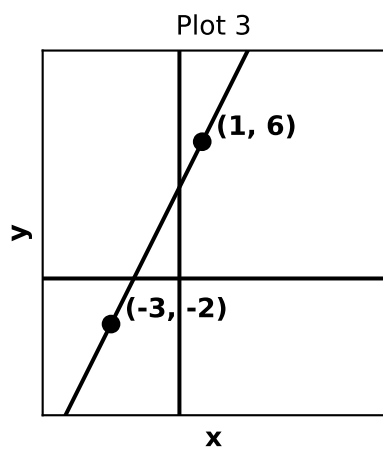
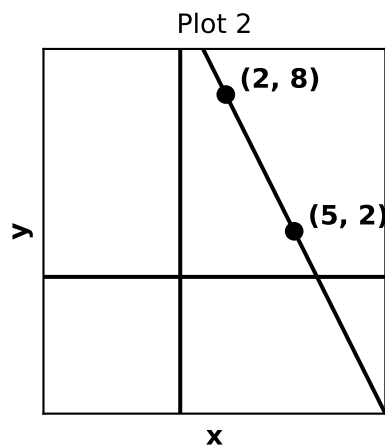
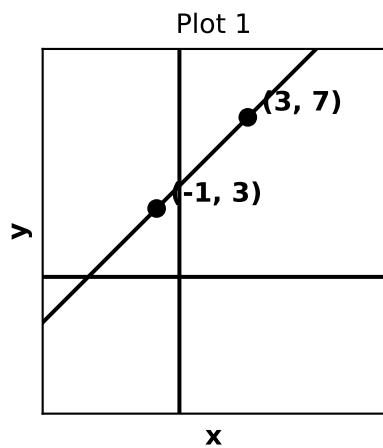
Example 19.11 (Activity: Finding the base). In Example 19.5 you found the slope of the line passing through the data table. Now compute the value of the base b to get a complete linear equation.

19. *The concept of slope*

Example 19.12 (Activity: Finding the base). In Example 19.6 you found the slope of the line passing through the data table. Now compute the value of the base b to get a complete linear equation.

19.6. Homework exercises

Exercise 19.1. For each plot below, compute the slope. Then find the equation of the line.



19. *The concept of slope*

Exercise 19.2. For each pair of points, find the equation of the line. Then draw a sketch of the line. Finally, determine the point where the line crosses the x axis.

1. $(1, 5)$ and $(3, 9)$

2. $(0, 4)$ and $(2, 0)$

3. $(-2, 1)$ and $(2, 5)$

4. $(1, 6)$ and $(4, 0)$

5. $(-1, -3)$ and $(3, 5)$

6. $(2, 8)$ and $(5, -1)$

19. *The concept of slope*

Exercise 19.4. It's firewood season, and if you don't own a chainsaw, you might need to rent one. At Taos Tool Rental, customers pay a rental fee and also pay per hour of use for a chainsaw.

- Marcus rents a chainsaw for 3 hours and is charged a total of \$35.
- Elena rents a chainsaw for 7 hours and is charged a total of \$55.

1. Determine the per-hour cost and also the rental fee.

2. Make a data table showing the total cost for up to 10 hours.

3. Make a plot showing cost on the vertical axis and number of hours on the horizontal axis.

4. How many hours can one rent the chainsaw before the total cost exceeds \$75?

20. Linear model synthesis

We now have assembled all our tools for constructing and studying linear models. In this section, we summarize the tools we have acquired, and practice them in a variety of ways. Always, we are trying to understand our models from three perspectives:

- Data table,
- Equation,
- Plot.

20.1. General features of the linear model

Our linear model is based on two numbers:

- b is the base value,
- m is the rate.

We can interpret these two numbers in all three perspectives.

Data table.

The base b tells us _____.

The rate m tells us _____.

We can compute the rate from the table using

$$m = \text{_____}$$

Equation.

The base and rate give us the equation _____.

Plot.

In terms of the plot, the base b gives us _____.

In terms of the plot, the rate m gives us _____.

20.2. Suppose we are given a scenario in words

We first want to check to determine whether or not a linear model is appropriate.

If a linear model is appropriate, then we can construct the data table, the equation, and the plot.

Example 20.1 (Activity: Scenario in words). The Taos Adobe Center charges \$3 per adobe brick, plus a \$50 packing fee.

1. Explain why the relationship between x = the number of adobe bricks and y = the fee charged is linear.
2. Construct a small data table for x and y .
3. Construct an equation relating x and y .
4. Make a plot of the linear relationship.

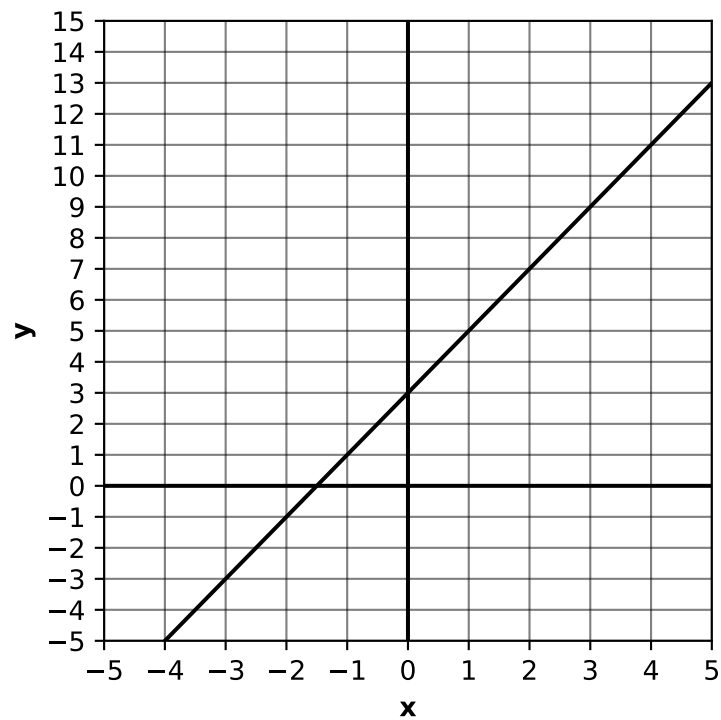
Example 20.2 (Activity: Scenario in words). Dr. Gutierrez is given a \$150 certificate for the Taos Cow. She rewards herself with a scoop of Rio Grande Cherry Ristra each Friday afternoon, costing her \$3.75 per week.

1. Explain why the relationship between x = the number of weeks and y = the balance on her gift card is linear.
2. Construct a small data table for x and y .
3. Construct an equation relating x and y .
4. Make a plot of the linear relationship.

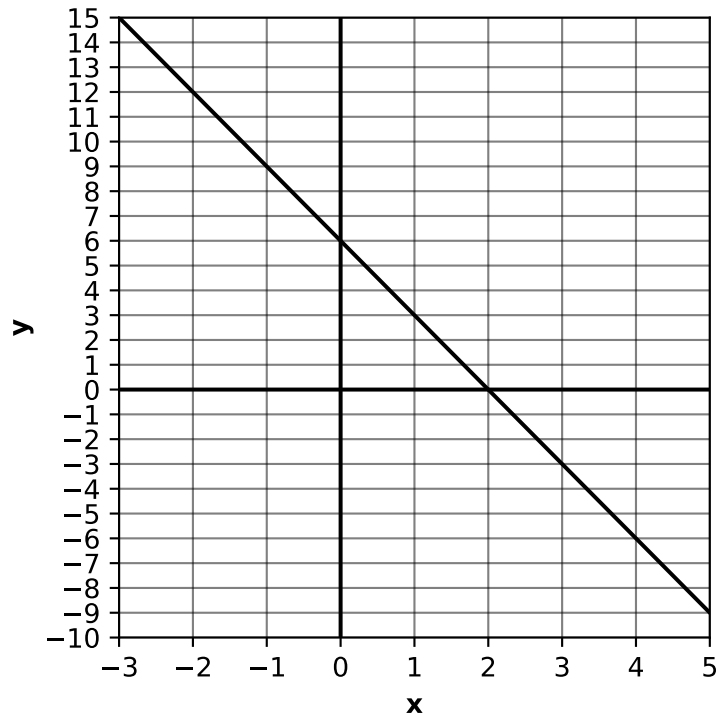
20.3. Suppose we are given the plot

If we are given the plot of a linear model, we can construct a data table and also construct the equation.

Example 20.3 (Activity: Given the plot). Construct a small data table and find the equation of the line.



Example 20.4 (Activity: Given the plot). Construct a small data table and find the equation of the line.



20.4. Suppose we are given an equation

If we are given the equation for a linear model, we can construct a data table and also construct the plot.

Example 20.5 (Activity: Given the equation). Consider the equation $y = 4x - 7$. Construct a small data table, and then sketch the plot of the line.

20.4. Suppose we are given an equation

Example 20.6 (Activity: Given the equation). Consider the equation $y = -3x + 12$. Construct a small data table, and then sketch the plot of the line.

20.5. Suppose we are given a data table

If we are given a data table for a linear model, we can construct the equation and we can construct the plot.

Example 20.7 (Activity: Given the data table). Here is a data table. Construct the equation for the linear model that matches the table, and sketch the plot.

x	y
0	5
1	9
2	13
3	17

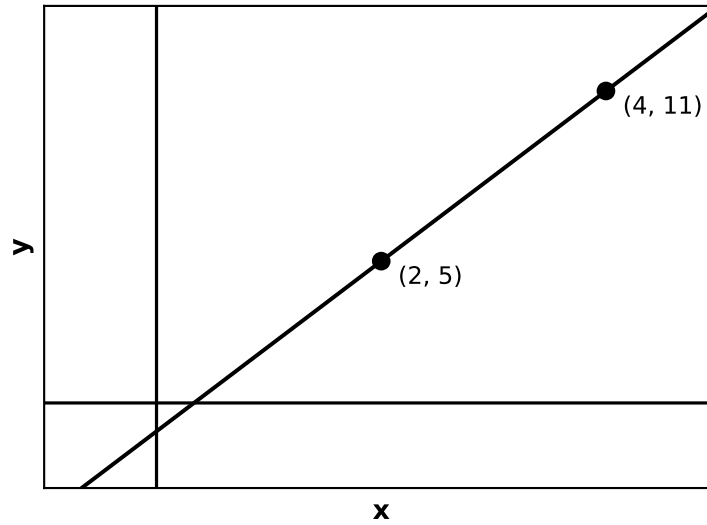
Example 20.8 (Activity: Given the data table). Here is a data table. Construct the equation for the linear model that matches the table, and sketch the plot.

x	y
1	7
2	10
3	13
4	16

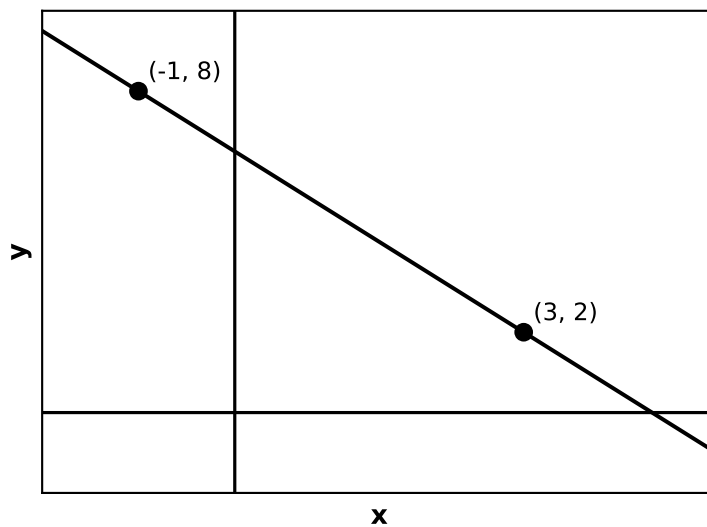
20. Linear model synthesis

Sometimes the data takes the form of given points on a graphic, rather than a formal data table.

Example 20.9 (Activity: Given the plot (no grid)). Find the equation of the line in the plot below. Then find the point where the line crosses the x axis.

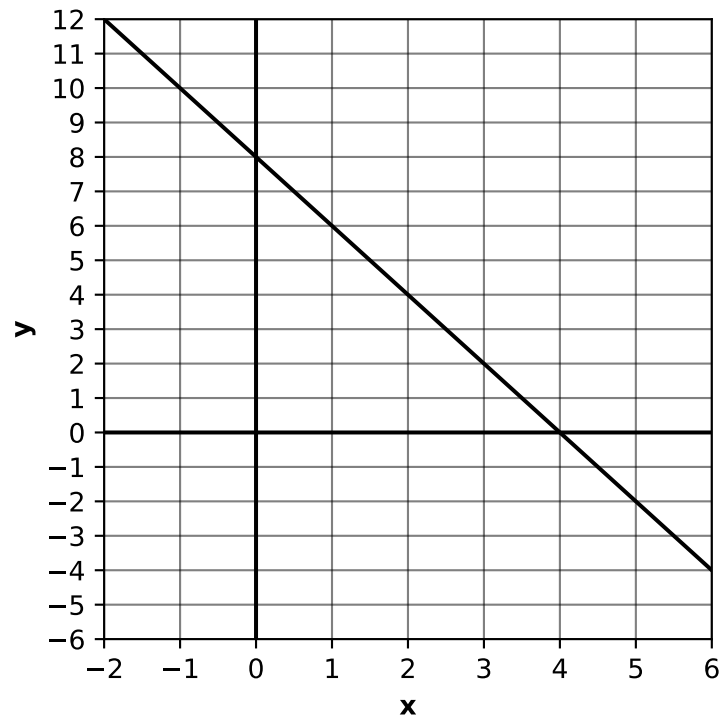


Example 20.10 (Activity: Given the plot (no grid)). Find the equation of the line in the plot below. Then find the point where the line crosses the x axis.

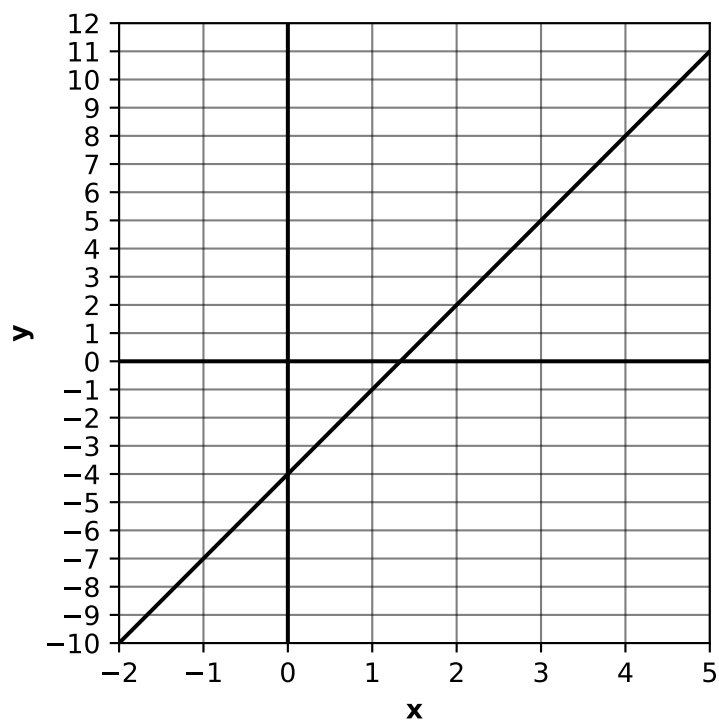


20.6. Homework exercises

Exercise 20.1. Make a data table, and find the equation, for the following line.



Exercise 20.2. Make a data table, and find the equation, for the following line.



20. *Linear model synthesis*

Exercise 20.3. Consider the equation $y = 6x + 2$. Construct a small data table, and then sketch the plot of the line.

Exercise 20.4. Consider the equation $y = 18 - 3x$. Construct a small data table, and then sketch the plot of the line.

20. *Linear model synthesis*

Exercise 20.5. Here is a data table. Construct the equation for the linear model that matches the table, and sketch the plot.

x	y
2	11
3	14
4	17
5	20

Exercise 20.6. Here is a data table. Construct the equation for the linear model that matches the table, and sketch the plot.

x	y
1	12
2	9
3	6
4	3

20. *Linear model synthesis*

Exercise 20.7. The Taos Lumber Yard charges \$8 per board of pine wood, plus a \$35 delivery fee.

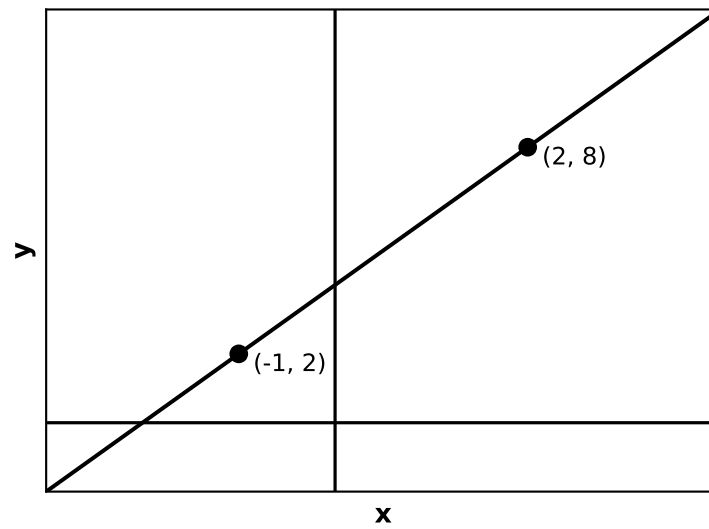
1. Explain why the relationship between x = the number of boards and y = the total cost is linear.
2. Construct a small data table for x and y .
3. Construct an equation relating x and y .
4. Make a plot of the linear relationship.

Exercise 20.8. At the beginning of the winter, the Taos County Road Division orders 1500 pounds of salt for treating roads. They estimate that they will use, on average, 200 pounds of salt per week.

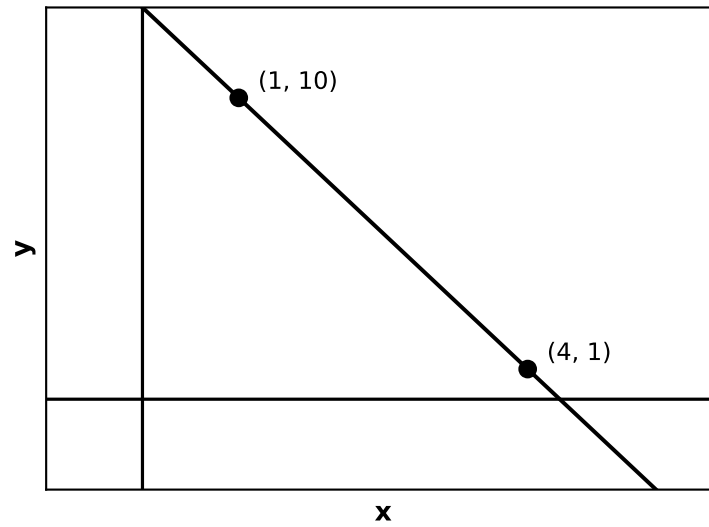
1. Explain why the relationship between $x =$ the number of weeks of winter and $y =$ the amount of salt remaining is linear.
2. Construct a small data table for x and y .
3. Construct an equation relating x and y .
4. Make a plot of the linear relationship.

20. Linear model synthesis

Exercise 20.9. Find the equation of the line in the plot below. Then find the point where the line crosses the x axis.



Exercise 20.10. Find the equation of the line in the plot below. Then find the point where the line crosses the x axis.



21. Linear practice

This section contains practice problems.

21.1. Linear scenarios

Example 21.1 (Activity: Scenario in words). The Rio Fernando Pizza Company charges customers a \$3.00 delivery fee, plus \$0.20 per mile, for delivering their famous green chile crust pizza. We want to describe how the cost of delivery depends on the delivery distance.

1. Explain why a linear model is appropriate here.
2. Construct the formula for the linear model.
3. Construct a small data table.
4. Sketch the plot of the linear model.
5. How much would it cost to get a pizza delivered to Tres Piedras, located 31 miles from the pizza company location?
6. How far away can the delivery be before the delivery cost exceeds \$10?

21. Linear practice

Example 21.2 (Activity: Scenario in words). The OBL Gravel Company charges \$50 per cubic yard of gravel, along with a delivery fee of \$75. We want to model how the cost depends on the amount of gravel being purchased.

1. Explain why a linear model is appropriate here.
2. Construct the formula for the linear model.
3. Construct a small data table.
4. Sketch the plot of the linear model.
5. How much would it cost to get 20 cubic yards of gravel?
6. How much gravel can be ordered before the cost exceeds \$500?

Example 21.3 (Activity: Scenario in words). At the end of the monsoon season, Elizabeth's cistern is full with 750 gallons of water. She uses an average of 40 gallons of water per day for drinking, cooking, cleaning, and bathing. We want to model the amount of water in her cistern over time.

1. Explain why a linear model is appropriate here.
2. Construct the formula for the linear model.
3. Construct a small data table.
4. Sketch the plot of the linear model.
5. How much water is left in the cistern after 2 weeks?
6. How long until only 100 gallons of water remain?

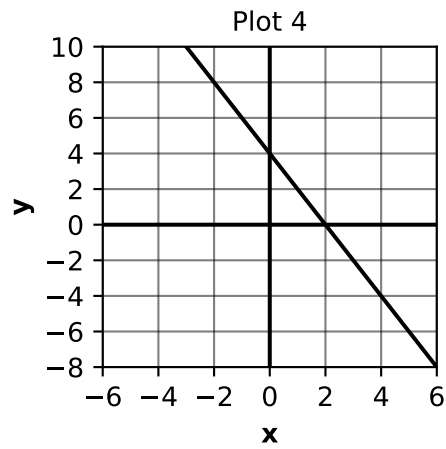
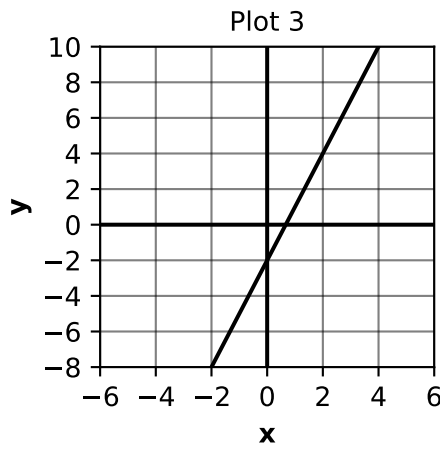
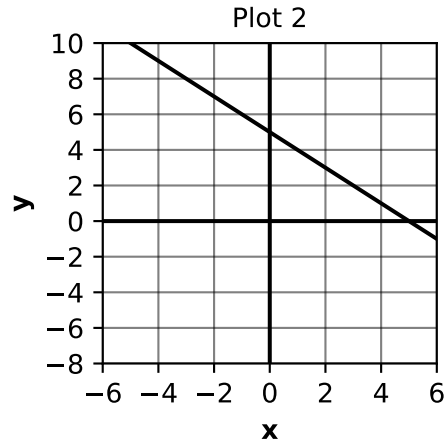
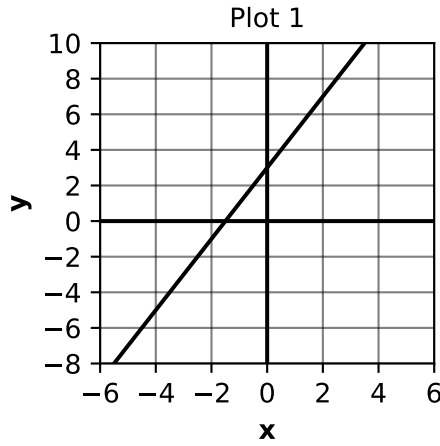
21. Linear practice

Example 21.4 (Activity: Scenario in words). Paul is saving up copies of the Taos News in a pile in his living room. Each newspaper is $\frac{1}{4}$ inch thick. In week 4 of the semester, the pile was 3 inches high. Each week, a new paper is added to the pile. We want to model the height of the pile in terms of the week of the semester.

1. Explain why a linear model is appropriate here.
2. Construct the formula for the linear model.
3. Construct a small data table.
4. Sketch the plot of the linear model.
5. How many papers were on the pile at the beginning of the semester?
6. How tall will the pile be in week 14 of the semester?
7. How long until the pile is a foot tall?

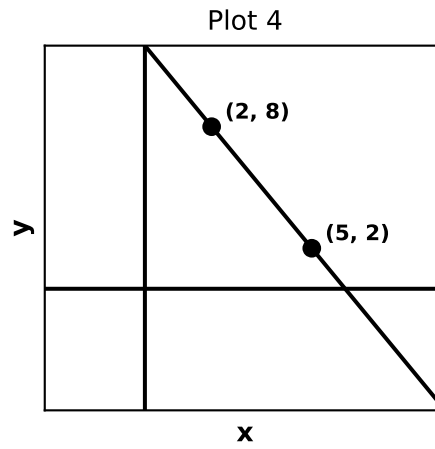
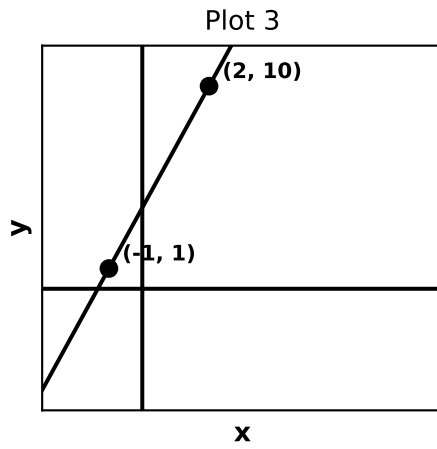
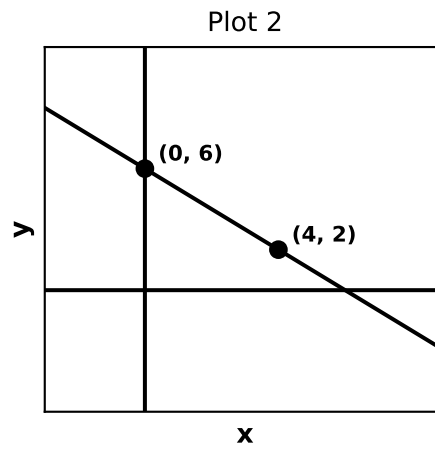
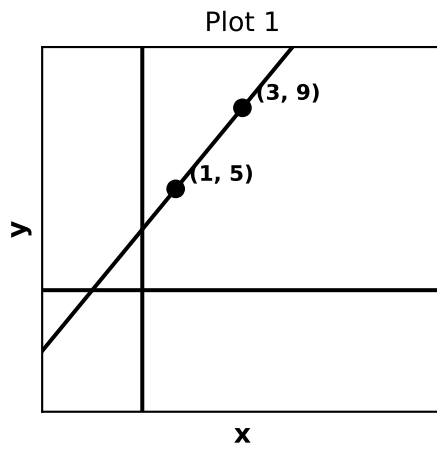
21.2. Interpreting linear plots

Example 21.5 (Activity: Interpreting a linear plot). For each line in the graphic below, find the base and rate, write down the equation, and determine where the line crosses the x axis.



21. Linear practice

Example 21.6 (Activity: Interpreting a linear plot). For each line in the graphic below, find the base and rate, write down the equation, and determine where the line crosses the x axis.



21.3. Interpreting linear equations

Example 21.7 (Activity: Interpreting a linear equation). Consider the line $y = 12 - 3x$.

1. What is the base? What is the rate?
2. Draw a sketch of the plot of the line.
3. Where does the line cross the x axis?

21. *Linear practice*

Example 21.8 (Activity: Interpreting a linear equation). Consider the line $y = 4x + 5$.

1. What is the base? What is the rate?
2. Draw a sketch of the plot of the line.
3. Where does the line cross the x axis?

Example 21.9 (Activity: Interpreting a linear equation). Consider the line $y = -\frac{1}{4}x + 6$.

1. What is the base? What is the rate?
2. Draw a sketch of the plot of the line.
3. Where does the line cross the x axis?

21. *Linear practice*

Example 21.10 (Activity: Interpreting a linear equation). Consider the line $y = \frac{3}{4}x - 9$.

1. What is the base? What is the rate?
2. Draw a sketch of the plot of the line.
3. Where does the line cross the x axis?

22. Project: Taos Bee Association

This project uses linear functions in order to analyze simple models for supply and demand.

In order to make it easier to understand these concepts, we study a hypothetical situation, where the *Taos Bee Association* is purchasing honey from local farmers and then selling the honey to tourists.

This project involves four tasks:

- Analyzing supply
- Analyzing demand
- Analyzing equilibrium
- Summarizing your analysis

22.1. Analyzing supply

We start by studying the honey supply. The Taos Bee Association plans to buy honey from a variety of farmers in Taos County. Economic theory tells us that if the Taos Bee Association pays a lower price for honey, then fewer people will want to sell them honey. But when the price is higher, then more people will be willing to sell them honey. The relationship between price and amount of honey are willing to sell is called a *supply curve*, even though in this case the relationship is actually a straight line.

We use these variables:

- x is the amount of honey local farmers are willing to sell, measured in hundreds of pounds. (Thus $x = 3$ means farmers are willing to sell 300 pounds of honey.)
- y is the price per pound of the honey. (Thus $y = 4$ means that the Taos Bee Association is paying farmers \$4 for a pound of honey.)

For the purposes of this project, we assume that the relationship between the amount of honey sold and the price is given by

$$y = 2 + 3x.$$

The first part of this task is to make a table of values for this formula. Be sure to label what x and y mean.

Second, you should make a plot of the equation. Be sure to label what the x and y axes mean.

Third, suppose that the Taos Bee Association is willing to buy honey for \$20 per pound. Based on the formula, how many pounds of honey should they expect to buy at this price?

22.2. Analyzing demand

We now analyze the honey demand. We imagine that the Taos Bee Association is selling honey to tourists who visit Taos County. Economic theory tells us that when the price of honey is high, then tourists will only buy a small amount of honey, but when the price of honey is low, then tourists will buy more honey.

We use these variables:

- x is the amount of honey tourists are willing to buy, measured in hundreds of pounds. (So $x = 3$ means that tourists are willing to buy 300 pounds of honey.)
- y is the price of a pound of honey. (So $y = 4$ means that honey is sold to the tourists for \$4 per pound.)

The Taos Bee Association did some market research and discovered that

- In order to sell 100 pounds of honey to tourists, the price can be set to \$40 per pound. *What values of x and y does this correspond to?*
- In order to sell 400 pounds of honey to tourists, the price can be set to \$28 per pound. *What values of x and y does this correspond to?*

The first part of this task is to make a table that includes the two data points from the market research. Also put these two points on a graph.

The second part of this task is to construct the linear equation $y = mx + b$ that connects the two data points. First, find the slope m , and then find the base value b .

The third part of the task is to draw the corresponding line.

Finally, answer these two questions:

- Suppose that the Taos Bee Association wants to sell 500 pounds of honey. What price should they set?
- Suppose the Taos Bee Association sets the price at \$4 per pound. How many pounds of honey will they be able to sell?

22.3. Analyzing the equilibrium

In an ideal world, the amount of honey that the farmers are willing to sell would be the same as the amount of honey that the tourists are willing to buy. We want to know: what price achieves this balance?

Think about how you might figure out what price achieves this balance. You can use any of our modeling approaches - table of values, plot, equation.

22.4. Summarizing your analysis

Finally, you should create a report that summarizes the results of your analysis. Your summary should include the following components:

- An introduction that explains the scenario and the questions that you addressed.
- The plot from your analysis of supply, along with some words explaining what the plot shows.
- The plot from your analysis of demand, along with some words explaining what the plot shows.
- The equilibrium value that you found, along with some words explaining how you found that value.

Please use complete sentences throughout your summary.

Part VII.

Exponential Models

23. The basic exponential model

The goal of this section is to introduce a type of nonlinear model called a *basic exponential model*. We start with three motivating examples.

Example 23.1 (Activity: Comparing two payment plans). Winners of the Taos County Xtreme Lottery receive an initial payment when they win, and then receive additional payments over a 30 week time period. They have a choice about how they want to receive their payments:

- **Option 1** is to receive an initial \$10,000 and then receive a \$10,000 check each week for the next 30 weeks.
- **Option 2** is to receive an initial payment of 1 penny (\$0.01), and then have the payment double each week during the 30 week time period.

Complete the data table below.

Week	Option 1 Payment	Option 2 Payment
0	\$10,000	\$0.01
1	\$10,000	$0.01 \cdot 2 = \$0.02$
2	\$10,000	$0.01 \cdot 2 \cdot 2 = \$0.04$
3		
4		
5		

23. *The basic exponential model*

Using the variables

- x is the number of weeks,
- y is the Option 2 payment (in dollars),

the formula for Option 2 is

$$y = \underline{\hspace{10em}}$$

Using your formula, what is the Option 2 payment in Week 30?

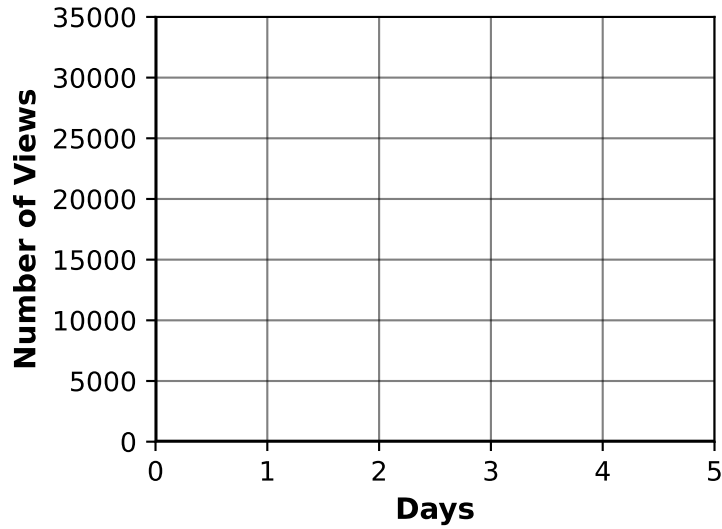
How does the Option 2 payment in Week 30 compare to the total of all payments in Option 1?

In what week does the Option 2 payment first exceed the Option 1 payment? (Hint: experiment with your formula.)

Example 23.2 (Activity: Mayor Dan’s PSA). Mayor Dan records a short public service announcement video and posts it to all the socials. Initially, the video gets only 10 views. But the video spreads! Each day, the number of views is 5 times the number of the previous day.

1. Make a data table showing the number of views each day for the first 5 days.

2. Plot your data on the following axes.



3. Using the variables

- x is the number of days since posting,
- y is the number of views,

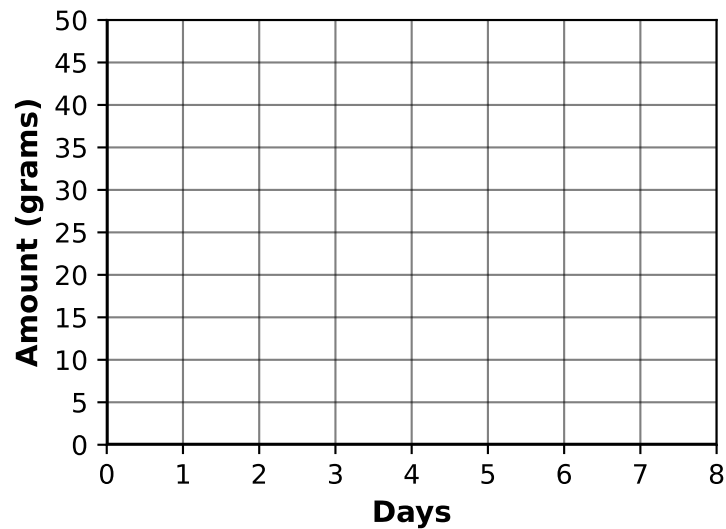
construct a formula for the number of views each day.

23. *The basic exponential model*

Example 23.3 (Activity: Diagnostic iodine). Iodine-123 is used for medical imaging, because it emits gamma rays as it decays away. The rate of decay for Iodine-123 is such that after 1 day, roughly 75% has decayed away, leaving only 25% of what was there the previous day. A lab starts the week with 48 grams of Iodine-123.

1. Make a data table showing the amount of Iodine-123 that remains over the course of 7 days.

2. Plot your data on the following axes.



3. Using the variables

- x is the number of days since the start of the week,
- y is the amount of Iodine-123 (in grams),

construct a formula for the amount remaining each day.

24. The exponential model framework

In the previous section we explored examples of exponential models and noted their common features. We now develop a general framework for describing and working with exponential models.

24.1. Parameters of the exponential model

Exponential models are determined by two numbers:

- A = the starting amount (also called the *initial value*)
- R = the *ratio* (also called the *multiplier*) by which the amount changes in one unit of time

Example 24.1. Recall the Mayor Dan example from the previous section. The video starts with 10 views and the number of views multiplies by 5 each day.

- $A = 10$ (the initial number of views)
- $R = 5$ (the daily multiplier)

Example 24.2. Recall the diagnostic iodine example from the previous section. The lab starts with 48 grams of Iodine-123 and each day 25% of the previous amount remains.

- $A = 48$ (the initial amount in grams)
- $R = \frac{1}{4}$ (the daily multiplier)

24.2. Discussion: Where do A and R appear?

In each example, A is the starting value. Where does A appear in the data table? Where does A appear in the plot?

In each example, R is the constant multiplier. Where does R appear in the data table? Where does R appear in the plot?

24. *The exponential model framework*

24.3. General formula

The general formula for the basic exponential model is

$$y = A \cdot R^x.$$

Take a moment to verify that this formula matches the Mayor Dan and iodine examples from the previous section.

24.4. Exponential increase and decrease

Example 24.3 (Activity: An increasing exponential model). Consider the exponential model $y = 10 \cdot 4^x$.

What is the starting amount for this model? What is the ratio?

Make a data table for this model.

Sketch the plot for this model.

What do you expect to happen to y when x is very large?

24. *The exponential model framework*

Example 24.4 (Activity: A decreasing exponential model). Consider the exponential model $y = 10 \cdot \left(\frac{1}{4}\right)^x$.

What is the starting amount for this model? What is the ratio?

Make a data table for this model.

Sketch the plot for this model.

What do you expect to happen to y when x is very large?

24.5. Discussion: increasing and decreasing models

Some exponential models are increasing and some are decreasing. How can we tell which is which by looking at the value of R ?

Sketch the plot of a typical increasing exponential model.

Sketch the plot of a typical decreasing exponential model.

24.6. Reading exponential models from data tables

So far we have built exponential models starting from real-world scenarios. We can also move in the other direction: given a data table, determine whether the model is exponential and, if so, identify A and R .

To check whether a data table represents an exponential model, compute the ratio of each output value to the previous one. If the ratio is the same every time, the model is exponential and that constant ratio is R .

Example 24.5 (Activity: Reading a data table). The following data table shows values of a model $y = f(x)$.

x	y
0	5
1	15
2	45
3	135
4	405

Compute the ratio of each consecutive pair of output values. Is the ratio constant?

Is this model linear, exponential, or neither? Explain.

Identify A and R , and write the formula $y = A \cdot R^x$.

24.7. Homework exercises

Exercise 24.1. A single yeast cell floats through the air and lands in a bowl of water and flour. Assume that yeast cells divide every minute.

1. Make a data table showing the number of yeast cells each minute for the first 8 minutes.
2. Explain why a basic exponential model is appropriate for this scenario.
3. Construct a plot of the data in your table.
4. Construct an equation that describes this scenario.
5. Use your equation to predict how many yeast cells there will be after 1 hour. Then predict how many there will be after 1 day.

24. *The exponential model framework*

Exercise 24.2. Wintertime is pothole season in Taos County, as the freeze/thaw cycle causes the roads to wear faster. At the beginning of the year, there are only 2 potholes on Paseo del Pueblo Sur. Every week, the number of potholes is three times larger than the previous week.

1. Make a data table showing the number of potholes during the first 8 weeks of winter.
2. Construct a plot of your data.
3. Build an exponential formula for the number of potholes in terms of the number of weeks.

Exercise 24.3. When dropped, the Amazing Bouncy Ball of Antonito bounces back up to half the height from which it was dropped. This continues for each bounce. Suppose the ball is dropped from the roof of the STEM building at a height of 24 feet.

1. Make a data table giving the height of the ball in terms of the number of bounces.
2. Make a plot of the data in your table.
3. Construct a formula for the height in terms of the number of bounces.
4. How high will the ball bounce on the 10th bounce?

24. *The exponential model framework*

Exercise 24.4. On a cold winter day in Taos, a math teacher accidentally leaves their coffee cup on the railing of their front porch. When first left behind, the coffee was 80 degrees Celsius. Each minute, the coffee is nine-tenths as warm as it was the previous minute.

1. Make a data table giving the temperature of the coffee each minute for a period of 5 minutes.
2. Make a plot of the data in your table.
3. Construct a formula for the temperature of the coffee in terms of minutes.
4. What will the temperature be after 15 minutes, when the math teacher goes to fetch it?

Exercise 24.5. Make a data table and a plot for the basic exponential model $y = 3 \cdot 2^x$.

24. *The exponential model framework*

Exercise 24.6. Make a data table and a plot for the basic exponential model $y = 24 \cdot \left(\frac{1}{2}\right)^x$.

Exercise 24.7. Consider the following scenarios. Determine whether a linear model or a basic exponential model would be more appropriate for each.

1. Each day, five soda cans are added to the recycle bin.
2. Each century, half of the radioactive material decays away.
3. Each semester, there are an additional 150 students attending UNM-Taos.
4. The cost of housing doubles every decade.
5. Each year, the price of a bowl of posole goes up by \$0.50.
6. Each year, the price of a breakfast burrito goes up by 5%.

24. *The exponential model framework*

Exercise 24.8. The following data table shows values of a model $y = f(x)$.

x	y
0	80
1	40
2	20
3	10
4	5

1. Compute the ratio of each consecutive pair of output values to confirm that this is an exponential model.
2. Identify A and R . Use them to write the formula $y = A \cdot R^x$ that matches the data table.
3. Use your formula to predict the value at $x = 6$.

25. Exponential models with percent change

In this section we analyze exponential models related to percent increase and decrease.

Example 25.1 (Activity: Fruit fly population). Due to an abandoned pile of bananas, fruit flies are growing rapidly in the kitchen. At the start of the week, there are 3 fruit flies. The number of fruit flies increases at a rate of 200% per day.

Increasing by 200% per day means that each day the population is $100\% + 200\% = 300\%$ of the previous day's population.

This means that after 1 day, the number of fruit flies is

$$\text{Flies Day 1} = 300\% \text{ of } 3 = \underline{\hspace{4cm}}$$

Use this information to complete the following data table.

Day	Fruit Flies
0	3
1	
2	
3	

What number do we multiply the fly population by to get the next day's population?

Using the variables

- x is the number of days,
- y is the number of flies,

construct a formula for the number of flies each day.

In terms of the standard formula $y = A \cdot R^x$:

- What is the value of A ?
- What is the value of R ?

How many flies does the model predict will be in the kitchen after 1 week?

25. Exponential models with percent change

25.1. The percent change framework

When a quantity changes by a fixed percent each time period, the multiplier R is determined by the percent rate r :

- If the quantity **increases** by r each period, then $R = 1 + r$.
- If the quantity **decreases** by r each period, then $R = 1 - r$.

The formula is then

$$y = A \cdot R^x,$$

where A is the starting amount and x is the number of time periods.

In the fruit fly example, the population increased by 200% per day, so $r = 2.00$ and $R = 1 + 2.00 = 3$. This confirms the formula $y = 3 \cdot 3^x$.

Example 25.2 (Activity: Fishing guides wage increase). The Association of Taos County Fishing Guides has negotiated a new contract. Currently, fishing guides are paid \$20 per hour. Under their contract, this amount will increase by 5% each year for the next 6 years. We want to model the hourly wage during the duration of the contract.

What is the starting amount A in this situation?

What is the rate r ? What are the time units?

What is the multiplier $R = 1 + r$?

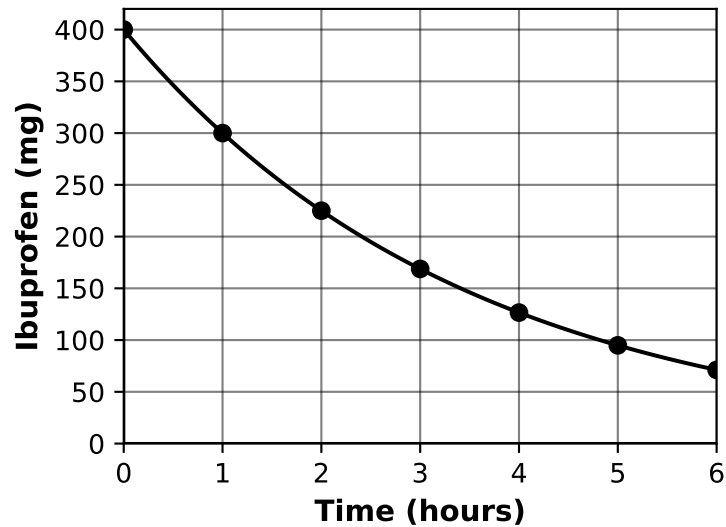
Using the variables $x = \text{years}$ and $y = \text{hourly wage (dollars)}$, write the equation for this model.

$$y = \underline{\hspace{10em}}$$

What will the guides' hourly wage be after 6 years?

25.2. Practice

Example 25.3 (Activity: Ibuprofen metabolism). A patient takes a 400 mg pill of ibuprofen. The following plot shows the amount of drug available to the body, which decreases as the drug is metabolized. Use the plot to identify A and R , and construct a formula for the amount of available ibuprofen (mg) in terms of time (hours). Use the value of R to determine the percent decrease per hour.



25. *Exponential models with percent change*

Example 25.4 (Activity: Mora County population). The 2020 population of Mora County was 4,189. Suppose that the county grows at 3% per year for the following decade.

1. Make a data table for the population over time.
2. Construct a formula for the population in terms of years since 2020.
3. Sketch a plot of your model.
4. According to your model, what will the population be in 2030? Label this point on your plot.

Example 25.5 (Activity: Truck depreciation). The value of equipment decreases over time; this is called *depreciation*. It is typical for pickup trucks to depreciate in such a way that they lose 10% of their value each year. Consider a new truck that costs \$50,000.

1. Make a data table showing the value of the truck over a five year period.
2. Construct a formula for the value of the truck over time.
3. Sketch a plot of the value over time.
4. What does your model predict the value will be 10 years after purchase? Label this point on your plot.

25.3. Homework exercises

Exercise 25.1. \$2,000 is put into a savings account that earns 4% interest per year, compounded annually.

1. Construct a data table showing the value of the account each year for 5 years.
2. Make a plot showing the data in your table.
3. Write down the formula for the value in terms of years.
4. How much money will be in the account after 5 years?

Exercise 25.2. A patient takes 500 mg of a medication. Each hour, 15% of the medication is eliminated from the bloodstream.

1. Construct a data table showing the amount of medication in the patient each hour.
2. Make a plot showing the data in your table.
3. Write down the formula for the medication in terms of hours.
4. How much medication remains after 6 hours?

25. *Exponential models with percent change*

Exercise 25.3. Consider the exponential model

$$y = 500 \cdot 2.5^x,$$

where x is measured in years.

1. What is the starting amount A for this model?
2. What is R ? What is the annual percent growth rate r ?
3. Draw a sketch of the plot for this model.
4. What will the amount be after 7 years? Show this point on your plot.

Exercise 25.4. A small town currently has a population of 1,000 residents. Two analysts make different projections for the town's growth over the next decade.

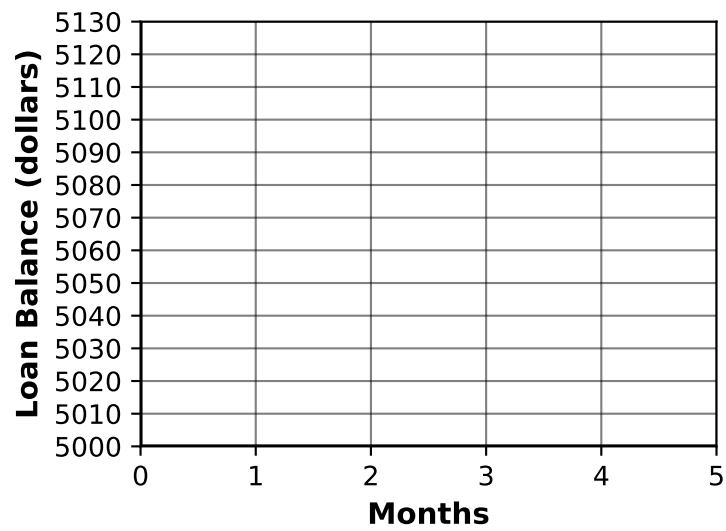
- **Analyst A** projects that the population will grow by 100 residents per year.
 - **Analyst B** projects that the population will grow by 10% per year.
1. Make a data table for each projection, showing the population for the first 6 years.
 2. Write a formula for each projection. Use $x =$ years and $y =$ population.
 3. Both projections give the same population after 1 year. Explain why this makes sense.
 4. Which projection predicts a larger population after 10 years? By how much?

26. *Compounded percent change*

1. Complete the data table for Genevieve's loan balance.

$x = \text{time (months)}$	$y = \text{loan balance (\$)}$
0	5,000
1	
2	
3	
4	

2. Plot the data from your table above.



3. Construct a formula that gives the loan balance in terms of months.

Use your formula to determine what the loan balance will be after 12 months.

26. Compounded percent change

26.1. The compounded percent change framework

When a percent rate r is applied N times per unit period (e.g., monthly within a year), the rate is divided among those N compounding periods. The multiplier for each period is

$$R = 1 + \frac{r}{N}.$$

With x measured in compounding periods, the formula is

$$y = A \cdot \left(1 + \frac{r}{N}\right)^x.$$

In the El Prado example, $A = 5000$, $r = 0.06$, $N = 12$, and x is months. So

$$y = 5000 \cdot \left(1 + \frac{0.06}{12}\right)^x = 5000 \cdot (1.005)^x.$$

Example 26.2 (Activity: Elk hunting guides wage increase). The Taos County Elk Hunting Guide Association has also negotiated a new contract. Currently, they are being paid \$20 per hour. Under their contract, this amount will increase by 5% each year, but with the increase computed quarterly (every 3 months). We want to model the hourly wage during the duration of the contract.

What is the starting amount A in this situation?

What is the annual rate r ?

What is N , the number of compounding periods per year? What time unit does each period represent?

What is the multiplier $R = 1 + \frac{r}{N}$?

Using x = number of quarters and y = hourly wage (dollars), write the equation for this model.

$$y = \underline{\hspace{10em}}$$

What will the guides' hourly wage be after 6 years?

26.2. Practice

Example 26.3 (Activity: Certificate of deposit). A local bank is offering a 4-year certificate of deposit (CD) at an annual rate of 3.3%. The interest is compounded monthly.

1. Suppose that \$1,000 is invested in this CD. Make a data table showing the value of the account during the first 4 months.
2. Construct a formula for the value in terms of months.
3. Sketch a plot of your model.
4. What will the value be at the end of the four year period?

26. *Compounded percent change*

26.3. Homework exercises

Exercise 26.1. A company purchases a skid steer for \$50,000. The equipment depreciates at a rate of 12% per year, calculated quarterly.

1. Construct a data table showing the value each quarter for the first 5 quarters.
2. Make a plot showing the data in your table.
3. Write down the formula for the value in terms of quarters.
4. What will the skid steer be worth after 3 years?

Exercise 26.2. \$8,500 is placed in a fund that grows at 7% per year, compounded quarterly.

1. Construct a data table showing the value of the account each quarter for the first 5 quarters.
2. Make a plot showing the data in your table.
3. Write down the formula for the value in terms of quarters.
4. How much will the investment be worth after 3 years?

26. *Compounded percent change*

Exercise 26.3. Consider the exponential model

$$y = 500 \cdot \left(1 + \frac{0.15}{12}\right)^x,$$

where x is measured in months.

1. What is the starting amount A for this model?
2. What is the annual rate r ? What is N ? How often is the rate being compounded?
3. Draw a sketch of the plot for this model.
4. What will the amount be after 7 years? Show this point on your plot.

27. Practice with exponential models

27.1. Exponential scenarios

Example 27.1 (Activity: Taos Ski Valley visitors). The Taos Ski Valley reports that the number of visitors has been growing at a steady rate of 4% per year. In 2020, the ski valley welcomed 275,000 visitors.

1. Make a data table showing the number of visitors for the five years following 2020.
2. Construct a formula for the number of visitors in terms of years since 2020.
3. Sketch a plot of your model.
4. According to your model, how many visitors will the ski valley welcome in 2030?

27. *Practice with exponential models*

Example 27.2 (Activity: Alfalfa hay spoilage). A rancher starts winter with 120 bales of alfalfa hay. Unfortunately, 40% of the remaining hay is lost to spoilage each month.

1. Make a data table showing the number of bales remaining each month for 5 months.
2. Construct a formula for the number of bales in terms of months.
3. Sketch a plot of your model.
4. How many bales remain after 8 months? Is that a practical amount?

Example 27.3 (Activity: Sourdough starter). A bread baker starts a sourdough culture with 10 grams of wild yeast. Under ideal conditions, the culture grows at a rate of 20% per day.

1. Make a data table showing the amount of culture each day for one week.

2. Construct a formula for the amount of culture in terms of days.

3. Sketch a plot of your model.

4. How much starter does the baker have after two weeks?

27. *Practice with exponential models*

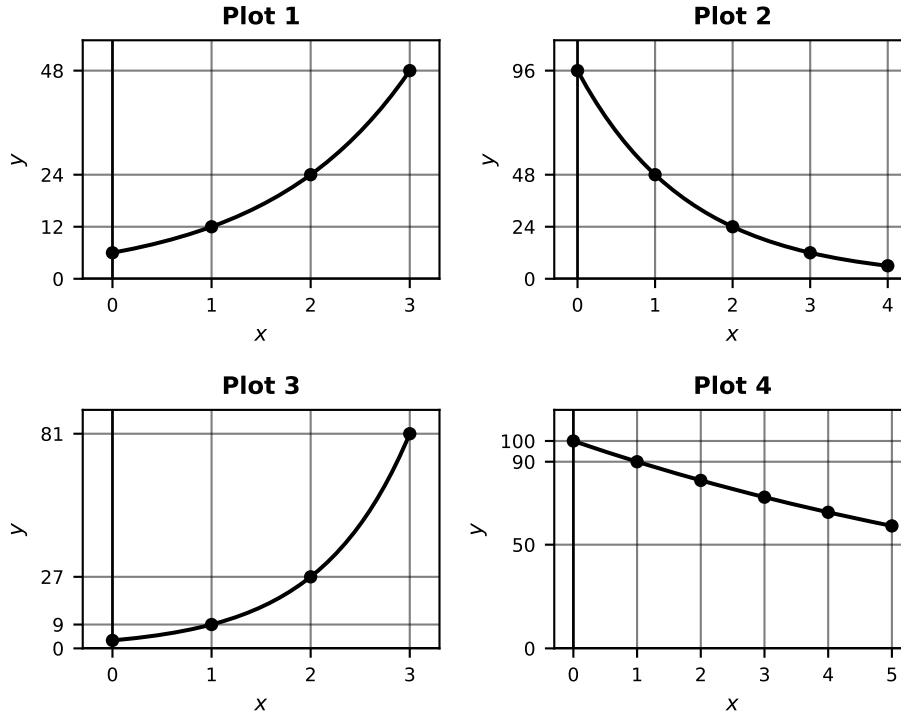
Example 27.4 (Activity: Quarterly investment). A UNM-Taos employee invests \$1,500 in a retirement fund earning 6% annually, compounded quarterly.

1. Make a data table showing the value of the investment each quarter for the first 4 quarters.
2. Construct a formula for the value in terms of quarters.
3. Sketch a plot of your model.
4. What will the investment be worth after 10 years?

27.2. Interpreting exponential plots

Example 27.5 (Activity: Reading exponential plots). Each plot below shows an exponential model $y = A \cdot R^x$. Two points on each curve are labeled.

For each plot, determine the values of A and R , and write the formula.



For each plot, fill in the table below.

	A	R	Formula
Plot 1			
Plot 2			
Plot 3			
Plot 4			

Which plots show increasing exponential models? Which show decreasing?

Example 27.7 (Activity: Monthly decay model). Consider the exponential model

$$y = 1200 \cdot (0.82)^x,$$

where x is measured in months.

1. What is the starting amount A ?
2. What is the multiplier R ? What is the monthly percent decrease rate r ?
3. Is this model increasing or decreasing? How can you tell from R ?
4. What will the amount be after 6 months?

27. Practice with exponential models

Example 27.8 (Activity: Quarterly compounding model). Consider the exponential model

$$y = 3500 \cdot \left(1 + \frac{0.045}{4}\right)^x,$$

where x is measured in quarters.

1. What is the starting amount A ?
2. What is the annual rate r ? What is N ? How often is the rate compounded?
3. What is the multiplier R ?
4. What will the amount be after 5 years?

27.4. Reading exponential models from data tables

Example 27.9 (Activity: Which tables are exponential?). Each table below shows the data table of some model. For each table, determine whether the model is linear, exponential, or neither. For any table that is exponential, identify A and R and write the formula.

Table A

x	y
0	4
1	12
2	36
3	108

Table B

x	y
0	100
1	85
2	70
3	55

Table C

x	y
0	200
1	160
2	128
3	102.4

27. Practice with exponential models

Example 27.10 (Activity: Which tables are exponential?). Each table below shows the data table of some model. For each table, determine whether the model is linear, exponential, or neither. For any table that is exponential, identify A and R , write the formula, and use the formula to predict the value at $x = 5$.

Table A

x	y
0	6
1	18
2	54
3	162

Table B

x	y
0	500
1	400
2	320
3	256

Table C

x	y
0	10
1	25
2	45
3	70

28. Project: Radioactive decay, half-lives, and C-14 dating

In this project we study exponential models for radioactive decay. The goal is to explore, understand, and use the concept of a *half-life* for radioactive isotopes.

28.1. Part 1: Iodine-131

Iodine-131 (called I-131 for short) is a radioactive isotope used in medical applications, especially for treating thyroid conditions. Any given quantity of I-131 will decay, losing roughly 8.3% per day.

Start your exploration with the following three tasks:

1. Suppose we start with 50 grams of I-131. Make a table of values showing how much remains each day for the first few days. Then construct a formula for the amount of I-131 remaining in terms of the number of days that has passed. After how many days has half of the Iodine-131 decayed away?
2. Suppose instead we start with 500 grams of I-131. Make a table of values showing how much remains each day for the first few days. Then construct a formula for the amount of I-131 remaining in terms of the number of days that has passed. After how many days has half of the Iodine-131 decayed away?
3. Suppose that we start with 1000 grams of I-131. Make a table of values showing how much remains each day for the first few days. Then construct a formula for the amount of I-131 remaining in terms of the number of days that has passed. After how many days has half of the Iodine-131 decayed away?

Based on the three tasks above, answer the following questions.

- Compare the three formulas you constructed. How are they similar? How are they different?
- In each of the three scenarios (starting with 50 grams, 500 grams, 1000 grams), compare how long it took for half of the Iodine-131 to decay away.

It should be the case that in each scenario, it took the same amount of time for half of the iodine to decay away. (If not, please go revisit your calculations before proceeding further!) This amount of time is called the *half-life* of Iodine-131.

28. Project: Radioactive decay, half-lives, and C-14 dating

i Link: Half-lives of common radioactive isotopes

Each radioactive isotope has a different value for its half-life, depending on the rate of decay. A list of common radioactive isotopes, and their half-lives, is available from the University of Alabama at Birmingham.

We can use the half-life of Iodine-131 as a unit of time. Suppose we start with 50 grams of I-131. Complete the following table.

Half-lives	Days	I-131 (grams)	Percent of I-131 remaining
0	0	50 g	100%
1		25 g	50%
2		12.5 g	
3			
4			

How many half-lives must pass until there is less than 1% of the original I-131 remaining? How many days is this? Given that I-131 is used in medical applications, what is the consequence of this time scale?

28.2. Part 2: Cesium-137

Cesium-137 (Cs-137 for short) is a radioactive isotope formed in nuclear reactors and other nuclear fission events that involve uranium. The half-life of Cs-137 is approximately 30 years.

Suppose we start with 24 grams of Cs-137. Complete the following table.

Half-lives	Years	Cs-137 (grams)	Percent of Cs-137 remaining
0	0	24 g	100%
1	30	12 g	50%
2			
3			
4			

How many half-lives must pass until there is less than 1% of the original Cs-137 remaining? How many years is this? Given that Cs-137 appear in radioactive waste, what are the practical consequences of this time scale?

Take a moment to compare the table you made for Iodine-131 and the table you made for Cesium-137. What parts of the tables are the same? What parts are different?

28.3. Part 3: Carbon-14

Carbon-14 (C-14) is an isotope of carbon created in earth's atmosphere by cosmic rays. These isotopes bind with oxygen to form a special type of carbon dioxide, which is in turn taken in by plants and animals. The result is that all living creatures have a certain (and known) amount of C-14 in them.

When a plant or animal dies, it stops taking in new C-14. Meanwhile, the Carbon-14 that exists in the plant or animal slowly decays with a half-life of roughly 5730 years.

Consider a tree that died many, many years ago. Complete the following table.

Half-lives	Years	Percent of C-14 remaining
------------	-------	---------------------------

Use your table (or an extension of your table) to address the following questions:

1. Suppose we find the remains of a tree that has 25% of the original C-14 in it. How long ago did the tree die?
2. Suppose we find the remains of a tree that has 5% of the original C-14 in it. Estimate how long ago the tree might have died.
3. Suppose less than 1% of the original C-14 remains in a tree sample. What can we say about how long ago the tree died?

28.4. Part 4: Exploration

Select one of the radioactive isotopes from the University of Alabama list above. Your goal is to construct a poster that contains the following features:

1. **An introduction to this isotope.** Using Wikipedia, or some other source, write a few sentences about the uses and/or origins of the isotope. *Be sure to write down the link to your source!*
2. **A computational example.** Suppose we have 5 grams of the isotope. Make a table showing half-lives, time, amount, and percent remaining. *Be sure to include the time units.*
3. **A qualitative discussion of the isotope.** Is the isotope long-lived or short-lived? Explain. What are the consequences of the lifespan of the isotope?

Any other fun or interesting elements you can add to your poster is appreciated!

